Bakshi, Gao, Panayotov: A theory of dissimilarity between stochastic discount factors

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SUMMARY

Construct a discrepancy measure — the Hellinger distance $H(m, m^*)$ — that

- has some appealing properties
- can be inferred from asset price data

Provide empirical evidence on $H(m, m^*)$

- from the cross section of currency option prices
- from the time series of currency returns
- from estimated SDFs using 'model-free' restrictions
- from structural models in international finance

Metrics allowing comparisons across classes of models are important.

QUESTIONS

1. Are the appealing properties utilized effectively?
2. Does the Hellinger distance provide insights different from other discrepancy measures?
3. Are quantitative implications clearly spelled out?
4. Have we learned something new?
5. Are there alternative interpretations of the discrepancy measure?

A NO-ARBITRAGE CONDITION

No-arbitrage condition in complete international markets

$$m_{t+1}\frac{S_{t+1}}{S_t} = m^*_{t+1}$$

- m_{t+1}, m^*_{t+1} domestic and foreign one-period SDFs
  - assume throughout that these are normalized to have conditional mean 1
- S_{t+1}/S_t exchange rate depreciation

APPROACH 1: HELLINGER DISTANCE FROM CURRENCY OPTIONS

Hellinger distance as a measure of dissimilarity of m_{t+1} and m^*_{t+1}

$$H\left(m_{t+1}, m^*_{t+1}\right) = 1 - E_t\left[\sqrt{m_{t+1} m^*_{t+1}}\right]$$

- see Schneider and Trojani (2015), Schneider (2017) for more extensive applications of Hellinger divergence in the study of market returns

A simple manipulation yields

$$H\left(m_{t+1}, m^*_{t+1}\right) = 1 - E_t\left[m_{t+1}\sqrt{\frac{m^*_{t+1}}{m_{t+1}}}\right] = 1 - E^Q_t\left[\sqrt{\frac{S_{t+1}}{S_t}}\right]$$

- RHS can be inferred from a cross-section of currency option prices

APPROACH 2: HELLINGER DISTANCE FROM EXCHANGE RATE TIME SERIES

Under log-normality of m_{t+1} and m^*_{t+1}

$$H\left(m_{t+1}, m^*_{t+1}\right) = 1 - \exp\left(-\frac{1}{8} Var_t\left[\log\frac{S_{t+1}}{S_t}\right]\right)$$

- when increments are independent, $Var_t\left[\cdot\right]$ can be proxied with daily variation

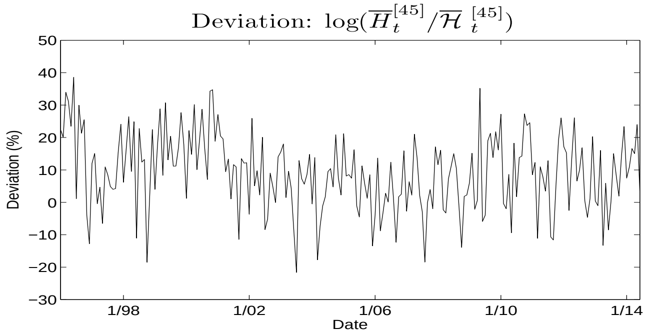
Use the two approaches to infer nonnormalities in SDFs.

RESULT: EVIDENCE OF NON-NORMALITIES

	$\sqrt{H} \times 100$				Percentiles				
	Mean	Std.	Min.	Max.	5th	25th	50th	75th	95th
A. Hellinger measures based on currency option prices									
$\overline{H}_t^{[45]}$	1.14	0.33	0.64	3.39	0.80	0.92	1.09	1.26	1.65
$\overline{H}_t^{US,[9]}$	1.12	0.34	0.59	3.34	0.73	0.94	1.08	1.24	1.64
B. Hellinger measures based on variance of currency returns									
$\overline{\mathcal{H}}_t^{[45]}$	1.05	0.36	0.55	3.90	0.69	0.84	0.99	1.13	1.66
$\overline{\mathcal{H}}_t^{US,[9]}$	1.03	0.37	0.48	3.53	0.61	0.83	0.97	1.15	1.77
C. Deviations									
$\log(\overline{H}_t^{[45]}/\overline{\mathcal{H}}_t^{[45]})$, %	10.3	12.8	-19.4	47.0	-10.5	1.2	10.0	18.2	31.9
$\log(\overline{H}_t^{US,[9]}/\overline{\mathcal{H}}_t^{US,[9]})$, %	11.1	14.4	-19.7	53.0	-10.0	1.1	10.5	19.9	38.4

- "We interpret the time-varying nature of the deviations as the contribution of stochastically-varying risk-neutral moments of currency returns"
- "Evidence of dissimilarity in higher moments of SDFs"

RESULT: EVIDENCE OF NON-NORMALITIES



- Evidence of non-normality or noise?
- Why is the Hellinger distance a superior measure of non-normality in currency returns?
- Why not study the distribution of $S_{t+1}/S_t = m^*_{t+1}/m_{t+1}$ directly?

RESULT: CROSS-COUNTRY DIFFERENCES

$\overline{H}_t^{i,[9]} - \overline{H}_t^{[45]}$	Mean	[95% CI]	Minimum	Maximum
Euro-zone (EU)	-0.17	[-0.21 -0.13]	-0.26	-0.12
United Kingdom (UK)	-0.09	[-0.10 -0.07]	-0.11	-0.05
Switzerland (SW)	-0.07	[-0.11 -0.03]	-0.14	0.02
Norway (NO)	-0.05	[-0.08 -0.03]	-0.11	0.00
Sweden (SD)	-0.03	[-0.06 -0.01]	-0.08	0.01
United States (US)	-0.01	[-0.08 0.05]	-0.15	0.09
Canada (CA)	0.00	[-0.04 0.04]	-0.07	0.08
Australia (AU)	0.04	[0.01 0.07]	-0.01	0.11
New Zealand (NZ)	0.12	[0.08 0.17]	0.05	0.22
Japan (JP)	0.23	[0.16 0.32]	0.09	0.43

- Are these differences quantitatively large?
- Interpretation of the magnitude of the Hellinger distance?

APPROACH 3: SDF FROM A MINIMUM DISPERSION PROBLEM

Infer m_{t+1} (and m^*_{t+1}) from a minimum discrepancy problem

$$\min_{m_{t+1}} E_t \left[m_{t+1} \log m_{t+1} \right]$$

subject to

$$1 = E_t \left[m_{t+1} R_{t+1} \right] \qquad\qquad 1 = E_t \left[m_{t+1} \frac{S_{t+1}}{S_t} R^*_{t+1} \right]$$

- Well-studied problem in the class of Cressie and Read (1984) discrepancies

Compare inferred m^*_{t+1}/m_{t+1} across countries with previous results

- use the Hellinger distance and other divergence measures

APPROACH 3: SDF FROM A MINIMUM DISPERSION PROBLEM

	T	Hellinger measure	Chi-squared measure	Volatility $\widetilde{n}$	Volatility $\widetilde{n}^*$
EU	191	0.81	0.096	105	104
UK	336	0.74	0.081	77	76
SW	336	0.89	0.107	84	84
CA	319	0.62	0.056	85	84
AU	336	0.97	0.127	74	70
NZ	309	0.98	0.138	76	70
JP	336	0.91	0.113	92	95

- Are the results 'close' to $H(m, m^*)$ obtained earlier?
- Hellinger and chi-squared measure are highly correlated.
- Why prefer the Hellinger measure?
    - Some computational simplicity but other than that?

APPROACH 4: MODEL COMPARISON

A) Verdelhan (2010); B) Lustig, Roussanov, Verdelhan (2014); C) Colacito, Croce (2011); D) time-varying disasters

	Models						Data			
	A	B			C	D				
		(i)	(ii)	(iii)						
δ		0.22	0.22	0.35			$\overline{H}_t^{[45]}$	$\overline{H}_t^{US,[9]}$	Smallest	Largest
δ^*		0.49	0.36	0.36					EU|SW	NZ|JP
Mean	2.93	1.60	1.26	1.07	1.21	1.31	1.14	1.12	0.54	1.56
							Bootstrap			
2.5 perc.	1.96	1.53	1.18	0.98	-	1.30	0.96	0.95	0.39	1.27
97.5 perc.	4.37	1.69	1.36	1.18	-	1.34	1.30	1.30	0.69	1.84

- Models differ in $H(m, m^*)$
    - Caveats: Calibrations to different periods, ...
- How much is economically significant?
- Should we calibrate models specifically to Hellinger distance?
    - What value added relative to other moments?

What if there is a friction in international financial markets?

$$m_{t+1}\frac{S_{t+1}}{S_t}\frac{F_{t+1}}{F_t} = m^*_{t+1}$$

- F_{t+1}/F_t represents evolution of shadow prices on financial constraints

Hellinger distance computed from currency options

$$\frac{S_{t+1}}{S_t} = \frac{m^*_{t+1}}{m_{t+1}}\frac{F_t}{F_{t+1}}$$

does not measure only the discrepancy in SDFs.

- but different ways of inferring m^*_{t+1}/m_{t+1} could inform us about F_{t+1}/F_t

SUMMARY

Creative, interesting paper

- model-free bounds have been informative about key restrictions on SDFs (Hansen and Jagannathan (1991))

Improve interpretation of the results

- which distribution characteristics does the Hellinger distance accentuate?
- quantitative interpretation of the distance

Can we utilize alternative approaches to think about frictions in international markets?