GEORGY CHABAKAURI, BRANDON YUEYANG HAN
CAPITAL REQUIREMENTS AND ASSET PRICES

Discussion by Jaroslav Borovička (NYU)

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- Brownian shock $\implies$ decentralization with a stock and a bond
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- a capital constraint $\implies$ agents cannot pledge (a part of) their future income

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Role of financial constraints

- Which constraints? How important are they? How do we distinguish them?

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- extra cherry on the cake: disaster insurance inducing jumps
  - delay term in the differential equation

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- Chabakauri and Han (2016): constraints and jumps
  - analogous representation, but economically much more interesting

State variable: adjusted ratio of martingal utilities

$$v_t = \ln \frac{(c_{At}/D_t)^{-\gamma_A}}{(c_{Bt}/D_t)^{-\gamma_B}} \doteq \frac{s_t^{-\gamma_A}}{(1-s_t)^{-\gamma_B}}$$

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Price-dividend ratio

$$\Psi(v) = \widehat{\Psi}(v; -\gamma_A)\, s(v)^{\gamma_A}$$

$\widehat{\Psi}(v;\theta)$ satisfies the ODE on $(\underline{v},\overline{v})$

$$\begin{aligned}
&\frac{\hat{\sigma}_v^2}{2}\widehat{\Psi}''(v;\theta)+(\hat{\mu}_v+(1-\gamma_A)\,\sigma_D\hat{\sigma}_v)\,\widehat{\Psi}'(v;\theta)-\\
&-\left(\lambda+\rho-(1-\gamma_A)\,\mu_D+\frac{(1-\gamma_A)\,\gamma_A}{2}\sigma_D^2\right)\widehat{\Psi}(v;\theta)+\\
&+\lambda\,(1+J_D)^{1-\gamma_A}\,\widehat{\Psi}'\left(\max\left\{\underline{v};v+\hat{J}_v\right\};\theta\right)+s\,(v)^{\theta}=0
\end{aligned}$$

with

$$\begin{aligned}
\hat{\mu}_v &= (\gamma_A-\gamma_B)\left(\mu_D-\frac{1}{2}\sigma_D^2\right)+\lambda-\lambda_B-\frac{\delta^2}{2}\\
\hat{\sigma}_v &= (\gamma_A-\gamma_B)\,\sigma_D+\delta\\
\hat{J}_v &= (\gamma_A-\gamma_B)\ln(1+J_D)+\ln\frac{\lambda_B}{\lambda}
\end{aligned}$$

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- But is this optimal?

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- E.g., low IES agent in a growing economy.

CAN THINGS CHANGE IN CONTINUOUS TIME?

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In fact, diverging leverage at the boundary seems to be necessary for the reflecting boundary.

- Otherwise volatility of v_t at the boundary would vanish.
- This would likely be inconsistent with a finite scale function (necessary for a reflecting boundary).

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The paper is very formal about treatment of boundary conditions.

- It would be useful to add a discussion of portfolio choices in the vicinity of the boundary.
- Compare with a discrete-time economy calculation.

Chabakauri (2013, RFS): Two stocks, heterogeneous RA, margin and leverage constraints

- positive relationship between leverage and stock return correlations and volatilities
- hump-shaped pattern of volatilities

AUTHORS' PREVIOUS WORK

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Chabakauri (2014): Heterogeneous EZ preferences, rare events

- excess stock return volatility, procyclical P/D ratios, countercyclical MPR when $IES > 1$

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3. How to think about policy experiments?
   - Which of these constraints represent structural restrictions?
   - Closed-form solutions are great for conducting such analysis.

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The paper contains relatively little comparison with authors' (and other) previous work.

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How does it happen? Unpledgeable future labor income

- E.g., *Cao (2014)*
- When agent depletes all financial wealth, she can still use flow of labor income to invest.
- Is this always true?

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On aggregate probably not.

- But maybe it is enough to look at the very rich.
- Better and better data available (*Matthieu Gomez (2015)*)

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SUMMARY

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    - Incorporates several appealing and carefully chosen components.
2. Analytical solution allows for convenient sensitivity analysis.
    - Do more of it!
3. Longer-term agenda should lead to comprehensive understanding of differences, similarities and relative importance of alternative constraints.
4. Empirical relevance of the wealth distribution dynamics