GEORGY CHABAKAURI, BRANDON YUEYANG HAN
CAPITAL REQUIREMENTS AND ASSET PRICES

Discussion by Jaroslav Borovička (NYU)
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A HETEROGENEOUS AGENT ECONOMY

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- Brownian shock $\rightarrow$ decentralization with a stock and a bond
- a jump shock with Poisson arrival rate $\rightarrow$ ‘insurance’ asset
- a capital constraint $\rightarrow$ agents cannot pledge (a part of) their future income
Closed form solution

- Impressive.
RESULTS

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Boundary behavior

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- Discrete- vs continuous-time economies.
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Role of financial constraints

- Which constraints? How important are they? How do we distinguish them?
Derive

- **law of motion for the state variable**: log ratio of marginal utilities
  - captures the current distribution of wealth in the economy
  - akin consumption share, wealth share, relative continuation values, Pareto share
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  - when the boundary is hit, difference in stochastic portfolio returns makes the wealth immediately reflect off the boundary
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- **extra cherry on the cake**: disaster insurance inducing jumps
  - delay term in the differential equation
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- **Chabakauri and Han (2016):** constraints and jumps
  - analogous representation, but economically much more interesting
State variable: adjusted ratio of martingal utilities

\[ v_t = \ln \frac{(C_{At}/D_t)^{-\gamma_A}}{(C_{Bt}/D_t)^{-\gamma_B}} = \frac{S_t^{-\gamma_A}}{(1 - S_t)^{-\gamma_B}} \]
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Price-dividend ratio

\[ \psi (v) = \hat{\psi} (v; -\gamma_A) s (v)^{\gamma_A} \]
$\hat{\Psi} \left( \nu; \theta \right)$ satisfies the ODE on $(\nu, \bar{\nu})$

$$\frac{\hat{\sigma}_V^2}{2} \hat{\Psi}'' \left( \nu; \theta \right) + \left( \hat{\mu}_V + \left( 1 - \gamma_A \right) \sigma_D \hat{\sigma}_V \right) \hat{\Psi}' \left( \nu; \theta \right) -$$

$$- \left( \lambda + \rho - \left( 1 - \gamma_A \right) \mu_D + \frac{\left( 1 - \gamma_A \right) \gamma_A}{2} \sigma_D^2 \right) \hat{\Psi} \left( \nu; \theta \right) +$$

$$+ \lambda \left( 1 + J_D \right)^{1-\gamma_A} \hat{\Psi}' \left( \max \left\{ \nu; \nu + \hat{J}_V \right\}; \theta \right) + s \left( \nu \right)^\theta = 0$$

with

$$\hat{\mu}_V = \left( \gamma_A - \gamma_B \right) \left( \mu_D - \frac{1}{2} \sigma_D^2 \right) + \lambda - \lambda_B - \frac{\delta^2}{2}$$

$$\hat{\sigma}_V = \left( \gamma_A - \gamma_B \right) \sigma_D + \delta$$

$$\hat{J}_V = \left( \gamma_A - \gamma_B \right) \ln \left( 1 + J_D \right) + \ln \frac{\lambda_B}{\lambda}$$
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- But is this optimal?
Timing

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- Portfolio choice. Portfolio cannot be excessively risky because we need $W_{t+\Delta t} \geq 0$. 

Given expected return on optimal portfolio, choose a saving rate $s \in [0; 1]$.

Saving decision

Clearly $s < 1$.

But do we know that $s > 0$? That must depend on parameterization.

E.g., low IES agent in a growing economy.
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In fact, diverging leverage at the boundary seems to be necessary for the reflecting boundary.

- Otherwise volatility of $v_t$ at the boundary would vanish.
- This would likely be inconsistent with a finite scale function (necessary for a reflecting boundary).
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The paper is very formal about treatment of boundary conditions.

- It would be useful to add a discussion of portfolio choices in the vicinity of the boundary.
- Compare with a discrete-time economy calculation.
Chabakauri (2013, RFS): Two stocks, heterogeneous RA, margin and leverage constraints

- positive relationship between leverage and stock return correlations and volatilities
- hump-shaped pattern of volatilities

Chabakauri (2015, JME): Heterogeneous beliefs and RA, various portfolio constraints

- borrowing and short-sale constraints decrease stock return volatility
- limited participation constraints increase volatilities

Chabakauri (2014): Heterogeneous EZ preferences, rare events

- excess stock return volatility, procyclical P/D ratios, countercyclical MPR when \( IES > 1 \)
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GENERAL TAKEAWAYS

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The paper contains relatively little comparison with authors’ (and other) previous work.
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WEALTH DISTRIBUTION AND SURVIVAL

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How does it happen? **Unpledgeable future labor income**

- E.g., *Cao (2014)*
- When agent depletes all financial wealth, she can still use flow of labor income to invest.
- Is this always true?
Distribution of wealth the only state variable.

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On aggregate probably not.

- But maybe it is enough to look at the very rich.
- Better and better data available (Matthieu Gomez (2015))
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4. Empirical relevance of the wealth distribution dynamics