Asset Pricing in the Frequency Domain: Theory and Empirics

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Discussed by Jaroslav Borovička

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Framework

- sources of (macro)economic risk

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- preferences as an aggregator of the risk sources (innovations representation)

\[ \Delta E_{t+1} [m_{t+1}] = -\Delta E_{t+1} \left( \sum_{k=0}^{\infty} z_k x_{t+1+k} \right) \]

- ‘myopic’ and ‘hedging’ demand
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- innovation to the SDF

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- correlation between \( \{z_k\} \) and \( \{g_k\} \)
Discrete-time Fourier transform

- representation in the frequency domain

\[ G(\omega) = \sum_{k=0}^{\infty} g_k e^{-i\omega k} \quad Z(\omega) = \sum_{k=0}^{\infty} z_k e^{-i\omega k} \]
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- Parseval’s theorem (frequency domain representation of a correlation)

\[ \sum_{k=0}^{\infty} z_k g_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} Z(\omega) G(\omega) \, d\omega \]

- \( Z(\omega) \) operates as a filter over macroeconomic risk \( G(\omega) \) at different frequencies
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This paper: What can we learn from the spectral decomposition of preferences \(Z(\omega)\)

- Estimate \(\{g_k\} (G(\omega))\) from data (VAR)
- Estimate different specifications for \(Z(\omega)\)
  - Some are linearizations of conventional preferences
  - Others have more statistical basis: aversion to risk at different frequencies
Goals

Why do we do all this?

1. Intuition
   - How do preferences load on different frequencies?

2. Estimation
   - Spectral decomposition cannot bring in any new information.
   - What if models are misspecified?
   - Estimating reduced form preference specification in the frequency domain.
Intuition

- Aversion to / preference for persistence
Estimation

Figure 4. Estimated spectral weighting functions for equities

All frequencies

- Bandpass basis
- Utility basis

Cycles longer than 5 years

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- Utility basis
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  - Group frequencies which the agent dislikes in a similar manner.
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  - Compare with tranching of mortgage-backed securities.
  - What if we take the model seriously and start fishing for cash flows which are underpriced/overpriced?
  - Very similar cash flows with frequencies concentrated around the steps should be priced quite differently.
  - **Security design**: spuriously attractive investment opportunities with very high Sharpe ratios.
Extensions

- Multiple periods
  - this paper: impact of low-frequency components on the one-period SDF
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  - *Alvarez–Jermann, Bakshi–Chabi-Yo*: bounds based on decomposition into permanent and transitory components
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M_t = \exp(\eta t) \hat{M}_t \frac{e(X_t)}{e(X_{t+1})}
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- Approximation errors
  - logarithms vs levels
  - loglinear approximation \( \Rightarrow \) bandpass filter \( \Rightarrow \) what happens to the SDF in levels?