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ESTIMATING THE VALUE OF INFORMATION

Discussion by Jaroslav Borovička (NYU)
November 2016
Value of being able to adjust choices to improve individual outcomes

- conventional measures: (abnormal) returns, profits
- based on *ex-post measurement*, including random ‘luck’
Value of being able to adjust choices to improve individual outcomes

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- based on *ex-post measurement*, including random ‘luck’

We need *ex-ante welfare measures*

- including dynamic effects
- portfolio allocation
- consumption/saving decisions
Value of being able to adjust choices to improve individual outcomes

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- based on *ex-post measurement*, including random ‘luck’

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- portfolio allocation
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Required

- a model of equilibrium prices, probabilities and stochastic discount factor
Massive! Covers key roles of the financial sector.

- Incentives to produce information
  - fundamental analysis, ...

- Incentives to hide/keep private information
  - insider trading, ...
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  - fundamental analysis, ...

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Existing work

- Large theoretical literature
- Some reduced-form empirical work
- Combining theory with detailed empirics is hard.
Ultimate goal

- general equilibrium asset pricing model of information production
- information production is often ignored — information is exogenous
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More modest goal

- Given a set of equilibrium prices ...
- ... compute the value of information
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More modest goal

- Given a set of equilibrium prices ...
- ... compute the value of information

Once obtained

- Think about incentives to produce information, etc.
Market equilibrium

- data-generating probability measure
- stochastic discount factor
- → prices of traded assets (Arrow–Debreu securities)
Market equilibrium

- data-generating **probability measure**
- stochastic discount factor
- \( \rightarrow \) **prices** of traded assets (Arrow–Debreu securities)

An individual atomistic agent

- receives an **imperfect signal** about next period state
- trades assets with this informational advantage
Market equilibrium

- data-generating probability measure
- stochastic discount factor
- $\Rightarrow$ prices of traded assets (Arrow–Debreu securities)

An individual atomistic agent

- receives an imperfect signal about next period state
- trades assets with this informational advantage

‘Partial equilibrium’ argument

- she is the only one who observes the signal
- her trades do not move equilibrium prices
- nobody else can infer anything from her behavior
Probabilistic environment

- Markov state of the world $z_t$
- Transition probability $p(z_{t+1}|z_t)$
Probabilistic environment

- Markov state of the world \( z_t \)
- Transition probability \( p(z_{t+1}|z_t) \)

Prices

- Equilibrium stochastic discount factor \( m(z_t, z_{t+1}) \)
- Prices of Arrow–Debreu securities

\[
q(z_{t+1}|z_t) = m(z_t, z_{t+1}) p(z_{t+1}|z_t)
\]
Details of the setup

Probabilistic environment

- Markov state of the world $z_t$
- Transition probability $p(z_{t+1}|z_t)$

Prices

- Equilibrium stochastic discount factor $m(z_t, z_{t+1})$
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$$q(z_{t+1}|z_t) = m(z_t, z_{t+1}) p(z_{t+1}|z_t)$$

Signal $s_t$ at time $t$ about state $z_{t+1}$: distribution $\alpha(s_t|z_{t+1})$

Bayes law: $p_\alpha(z_{t+1}|s_t, z_t) = \frac{\alpha(s_t|z_{t+1}) p(z_{t+1}|z_t)}{p_\alpha(s_t|z_t)}$
Atomistic agent optimally makes

- a consumption-saving decision $C_t$
- a portfolio choice $W_t$
Atomistic agent optimally makes

- a consumption-saving decision $c_t$
- a portfolio choice $w_t$

To maximize utility (Epstein–Zin preferences)

\[
V (a_t, z_t, s_t) = \max_{c_t, w_t} \left\{ (1 - \beta) c_t^{1-\rho} + \beta E_t \left[ V (a_{t+1}, z_{t+1}, s_{t+1})^{1-\gamma} \right]^{1-\frac{\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}
\]

Subject to

\[
a_{t+1} = (a_t - c_t) (w_t \cdot R_t)
\]
Atomistic agent optimally makes

- a consumption-saving decision $c_t$
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To maximize utility (Epstein–Zin preferences)

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subject to

$$a_{t+1} = (a_t - c_t)(w_t \cdot R_t)$$

Compare value functions with and without signal being available.
EMPIRICAL IMPLEMENTATION

**Inputs**

- choice of the Markov state $z_t$ (*S&P 500 index*)
- events with informative signals $s_t$ (*releases of macro indicators*)
- prices of Arrow–Debreu securities $q(z_{t+1}|z_t)$ (*from option prices*)
Prices of Arrow–Debreu securities (state prices) $q(Z_{t+1}|Z_t)$

- from S&P 500 index European options (Breeden, Litzenberger (1978))
- smoothing using a parametric Carr, Wu (2010) model
- discretization of the state space as in Ross (2015)

Pretty standard in the literature
**ESTIMATION OF STATE PRICES**

**Prices of Arrow–Debreu securities** (state prices) \( q (Z_{t+1}|Z_t) \)

- from S&P 500 index European options (*Breeden, Litzenberger (1978)*)
- smoothing using a parametric *Carr, Wu (2010)* model
- discretization of the state space as in *Ross (2015)*

Pretty standard in the literature

Use asset price data

- before the release of macro indicator \( q (Z_{t+1}|Z_t) \)
- after the release of macro indicator \( q (Z_{t+1}|Z_t, S_t) \)
This is a hard problem!

- strong assumptions needed $\implies$ authors try two alternatives
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- **strong assumptions needed** \(\implies\) authors try two alternatives

1) **parametric specification** of preferences/stochastic discount factor

- CRRA/Epstein–Zin preferences, iid growth and returns

\[
m(z_t, z_{t+1}) \propto \exp(-\gamma R_p(z_{t+1}))
\]
This is a hard problem!

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1) **Parametric specification** of preferences/stochastic discount factor

- CRRA/Epstein–Zin preferences, iid growth and returns

$$m (z_t, z_{t+1}) \propto \exp ( -\gamma R_p (z_{t+1}) )$$

2) **Nonparametric specification**

- separable stationary marginal utility ([Ross (2015)](https://example.com))

$$m (z_t, z_{t+1}) = \delta \frac{\tilde{m} (z_{t+1})}{\tilde{m} (z_t)}$$

- solution to a Perron–Frobenius problem
Infer the transition probabilities $p(z_{t+1}|z_t)$ and $p_\alpha(z_{t+1}|s_t, z_t)$ as $\frac{q(z_{t+1}|z_t)}{m(z_t, z_{t+1})}$

- using price data before and after the information release.
Infer the transition probabilities \( p(z_{t+1}|z_t) \) and \( p_\alpha (z_{t+1}|s_t, z_t) \) as

\[
\frac{q(z_{t+1}|z_t)}{m(z_t, z_{t+1})}
\]

- using price data before and after the information release.

Construct two value functions

- **Uninformed agent**: Trades at prices \( q(z_{t+1}|z_t) \) under beliefs \( p(z_{t+1}|z_t) \)
- **Informed agent**: Trades at prices \( q(z_{t+1}|z_t) \) under beliefs \( p_\alpha (z_{t+1}|z_t, s_t) \)

Compare value functions to infer the value of information
1. **Sensitivity of results** to changes in preference parameters
   - How to pin down these preference parameters?

2. Estimation of the **data-generating probability**
   - Recovery?
   - Nonparametric methods
   - Other sources of information

3. **Equilibrium considerations**
Results heavily depend on preference parameters (we don’t agree on them!)

- **low risk aversion / high IES** allow to take advantage of knowing the signal
Results heavily depend on preference parameters (we don’t agree on them!)

- **low risk aversion** / **high IES** allow to take advantage of knowing the signal

**How to discipline the preference parameters?**

- portfolio positions, volume of trade
- it would be useful to see implied portfolio decisions and saving choices
Arrow–Debreu prices $q(z_{t+1}|z_t)$ reflect risk-adjusted probabilities of alternative states.

- For some welfare questions, these are the right probabilities to look at.
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- Alvarez, Jermann (2004) — use asset prices to infer the cost of business cycles
  - see also Otrok (2001), Croce (2013)
- Feldman et al. (2016) — use \( q(z_{t+1}|z_t) \) to study the welfare impact of monetary policy
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The data-generating probability measure is needed to infer informativeness of the signal.
Preferences and beliefs cannot be separately identified from asset prices without additional assumptions.

- Borovička, Hansen, Scheinkman (2016)
Preferences and beliefs cannot be separately identified from asset prices without additional assumptions.

- Borovička, Hansen, Scheinkman (2016)— given arbitrary $h(z_t, z_{t+1})$ with mean one:

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q(z_{t+1}|z_t) = m(z_t, z_{t+1}) p(z_{t+1}|z_t)
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- Borovička, Hansen, Scheinkman (2016) — given arbitrary $h(z_t, z_{t+1})$ with mean one:

$$q(z_{t+1}|z_t) = m(z_t, z_{t+1})p(z_{t+1}|z_t) = \frac{m(z_t, z_{t+1})}{h(z_t, z_{t+1})} [h(z_t, z_{t+1})p(z_{t+1}|z_t)]$$

$$\tilde{m}(z_t, z_{t+1})\tilde{p}(z_{t+1}|z_t)$$

- $\tilde{p}(z_{t+1}|z_t)$ is a valid probability measures
- $\tilde{m}(z_t, z_{t+1})$ is a valid stochastic discount factor
What can be done?

- pick form of preferences $\implies$ determine $m(z_t, z_{t+1})$
- determine

$$p(z_t, z_{t+1}) = \frac{q(z_{t+1}|z_t)}{m(z_t, z_{t+1})}$$
What can be done?

- pick form of preferences \( \implies \) determine \( m(z_t, z_{t+1}) \)
- determine

\[
p(z_t, z_{t+1}) = \frac{q(z_{t+1}|z_t)}{m(z_t, z_{t+1})}
\]

- authors try two forms
  - recursive/CRRA preferences with a given relative risk aversion \( \gamma \) and iid growth
    
    \[
m(z_t, z_{t+1}) \propto \exp (-\gamma R_p(z_{t+1}))
    \]
  - separable preferences in stationary environment (Ross (2015))
    
    \[
m(z_t, z_{t+1}) = \delta \frac{\bar{m}(z_{t+1})}{\bar{m}(z_t)}
    \]
Figure 5: Prior vs. Posterior Probabilities around Unemployment Releases
Table 2: Estimated Value of Information as Percent of Wealth: Power U Recovery

<table>
<thead>
<tr>
<th>Event</th>
<th>$RRA = 5 = 1/EIS$</th>
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<th>$RRA = 5, EIS = 0.90$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\Omega}$</td>
<td>$se(\hat{\Omega})$</td>
<td>$\hat{\Omega}$</td>
</tr>
<tr>
<td>GDP</td>
<td>1.23 (0.51)</td>
<td>1.23</td>
<td>94.00 (3.90)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>1.13 (0.41)</td>
<td>1.13</td>
<td>89.94 (4.72)</td>
</tr>
<tr>
<td>Jobless Claims</td>
<td>1.51 (0.27)</td>
<td>1.51</td>
<td>93.79 (2.30)</td>
</tr>
<tr>
<td>Pre-FOMC</td>
<td>1.53 (0.98)</td>
<td>1.53</td>
<td>95.77 (5.18)</td>
</tr>
<tr>
<td>FOMC</td>
<td>0.84 (0.68)</td>
<td>0.84</td>
<td>95.73 (3.96)</td>
</tr>
<tr>
<td>Mortgage App.</td>
<td>2.49 (0.59)</td>
<td>2.49</td>
<td>96.51 (2.18)</td>
</tr>
<tr>
<td>Consumer Comfo.</td>
<td>2.10 (0.46)</td>
<td>2.10</td>
<td>96.54 (2.04)</td>
</tr>
</tbody>
</table>

Table 3: Estimated Value of Information as Percent of Wealth: Ross Recovery

<table>
<thead>
<tr>
<th>Event</th>
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</tr>
<tr>
<td>GDP</td>
<td>2.72 (0.58)</td>
<td>2.72</td>
<td>97.40 (2.48)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>3.08 (0.47)</td>
<td>3.08</td>
<td>99.16 (0.45)</td>
</tr>
<tr>
<td>Jobless Claims</td>
<td>2.75 (0.30)</td>
<td>2.75</td>
<td>97.82 (1.34)</td>
</tr>
<tr>
<td>Pre-FOMC</td>
<td>3.26 (1.20)</td>
<td>3.26</td>
<td>98.78 (1.60)</td>
</tr>
<tr>
<td>FOMC</td>
<td>3.19 (0.91)</td>
<td>3.19</td>
<td>98.79 (1.06)</td>
</tr>
<tr>
<td>Mortgage App.</td>
<td>4.01 (0.74)</td>
<td>4.01</td>
<td>98.20 (1.18)</td>
</tr>
<tr>
<td>Consumer Comfo.</td>
<td>3.79 (0.52)</td>
<td>3.79</td>
<td>99.24 (0.72)</td>
</tr>
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</table>
Alternatives

- **Time series information** on transition probabilities
- **Surveys?** Do subjective beliefs correspond to rational expectations?
  - Bhandari, Borovička, Ho (2016)
- **Nonparametric estimation** of the pricing kernel
  - Christensen (2015, 2016)
The paper is about individual decisions of an agent vis-à-vis given equilibrium prices.

- No equilibrium interaction.
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- No equilibrium interaction.

**What other questions would we like to answer?**

1. What are the incentives to create information for private/public use in a **competitive environment**?

2. What are the incentives to share/reveal information?
   - price impact, diffusion of information
   - protection of information rents
Moving away from the assumption of exogenous and common information is crucial.

- The right step in this direction.
- Quantitative empirical implementation (very hard to do).
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**Next steps?**

- Extensions see above.
- *More discipline on estimating the SDF* $\Rightarrow$ inference of the data-generating probability measure.