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INFORMED TRADING AND INTERTEMPORAL SUBSTITUTION: THE LIMITS OF THE NO-TRADE THEOREM

Discussion by Jaroslav Borovička (NYU)
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No-trade theorem(s) (Milgrom and Stokey (1982) and subsequent extensions) show that

- when preferences are separable
- and we start from a Pareto-optimal allocation
No-trade theorem(s) (*Milgrom and Stokey* (1982) and subsequent extensions) show that

- when preferences are separable
- and we start from a Pareto-optimal allocation

then subsequent release of (private or public) information cannot lead to retrading.
Separable preferences

\[ U^i = E \left[ \sum_{t=0}^{\infty} \beta^t u^i \left( c_t^i; \theta_t \right) \right] \]
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Planner's problem

\[
\max \sum_{i} \lambda^i U^i \\
\text{subject to } \sum_{i} c^i_t \leq Y_t \left( \theta^t \right)
\]

- optimal consumption allocation only depends on \( Y_t \left( \theta^t \right) \) (and \( u^i (\cdot; \theta_t) \))
- not on any other aspects of the history or future
Separable preferences

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First-order conditions

\[
\sum_i \lambda^i (u')' (c_t^i; \theta_t) = \mu_t \left( Y_t \left( \theta^t \right) \right)
\]

\( \mu \) is the L.M. on the constraint

- completely static, separable problem.
Why no retrading after release of information?

Imagine release of additional private or public information $x^{i,t}$. 
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Imagine release of additional private or public information $x^{i,t}$.

Potential retrading would have lead to an allocation that depends

- not only on $\theta_t$ and $Y_t(\theta^t)$
- but also on other information $c_t^i = c_t^i(\theta_t, Y_t, x^{i,t})$
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but then each risk-averse agent would prefer $E \left[ c^i_t \mid \theta_t, Y_t \right]$ to $c^i_t$

- which is also feasible
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which could have been chosen by the planner

- but wasn’t $\implies$ contradiction
Incomplete markets

· starting from a non-Pareto optimal allocation $\implies$ re-trading possible
· (note: this is different from dynamically complete markets)
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Non-separable preferences

- this paper
- habit formation, Epstein–Zin, ...
An agent has risk-aversion-dominating preferences when, \( \forall t \) and \( \forall C \)

\[
C = (c_0, c_1, \ldots, c_t, \ldots) \preceq (c_0, c_1, \ldots, E[C_t \mid \theta_t], \ldots)
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Notice that this is exactly what is needed in the no-trade theorem

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It does not need to hold for non-separable preferences anymore

- $c_t$ impacts marginal utility of consumption in other states and periods
- it may make sense to correlate $c_t$ with consumption in other periods
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It does not need to hold for non-separable preferences anymore

- \( c_t \) impacts marginal utility of consumption in other states and periods
- it may make sense to correlate \( c_t \) with consumption in other periods
- additional information (e.g., about future states) can lead to retrading
Non-separable, recursive (dynamically consistent) preference structure.

\[ U_t = \left[ c_t^{1-\rho} + \beta E \left[ U_{t+1}^{1-\gamma} \mid \mathcal{F}_t \right]^{\frac{1-\rho}{1-\gamma}} \right] \]

- \( \gamma \) risk aversion, \( \rho \) IES, \( \beta \) time preference
- An example of the Kreps–Porteus recursive preferences
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Kreps–Porteus: preference for timing of information

- The above is a special case of the aggregator (after a transformation)

$$V_t = f(c_t, E[V_{t+1} \mid \mathcal{F}_t])$$
Kreps–Porteus recursive preferences

\[ V_t = f(c_t, E[V_{t+1} | \mathcal{F}_t]) \]

- when \( f \) is concave in its second argument then

\[ f(c_t, E[V_{t+1} | \mathcal{F}_t]) \leq E[f(c_t, V_{t+1}) | \mathcal{F}_t] \]

\( \implies \) preference for early resolution of uncertainty
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  \( \implies \) preference for **late resolution of uncertainty**
Recall the concept of the no-trade theorem experiment

First open an ex-ante complete market where period $t$ consumption claims can be traded conditional on the history $\theta^t$.

- Filtration $\{\mathcal{F}_t\}$.
- Agents will trade to a Pareto-optimal allocation
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Then provide additional (private or public) information about which state will be realized.

- Filtration $\{\mathcal{F}^*_{t}\}$.
- Under assumptions of the no-trade theorem, no re-trading.
- All trade-relevant information already summarized in $(\theta_t, \gamma_t(\theta^t)) \in \mathcal{F}_t$. 
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In a dynamic environment, we need to specify how we got to the initial Pareto optimal allocation.
Two period example

1. Let the agents trade in a complete state-contingent market with information \( \{ F_t \} \).

\[
V_1 = \left[ c_1^{1-\rho} + \beta E \left[ c_2^{1-\gamma} \mid F_1 \right]^{\frac{1-\rho}{1-\gamma}} \right]
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Two period example

1. Let the agents trade in a complete state-contingent market with information \( \mathcal{F}_t \).

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V_1 = \left[ c_1^{1-\rho} + \beta E \left[ c_2^{1-\gamma} | \mathcal{F}_1 \right]^{1-\frac{\rho}{1-\gamma}} \right]
\]

2. After trading, \textbf{unexpectedly} announce new information \( \mathcal{F}_t^* \)

\[
\mathcal{F}_1^* = \mathcal{F}_2.
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3. New preferences

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2. After trading, unexpectedly announce new information $\{\mathcal{F}_t^*\}$

\[ \mathcal{F}_1^* = \mathcal{F}_2. \]

3. New preferences

\[ V_1^* = \left[ c_1^{1-\rho} + \beta E \left[ c_2^{1-\gamma} \mid \mathcal{F}_1^* \right]^{\frac{1-\rho}{1-\gamma}} \right] = \left[ c_1^{1-\rho} + c_2^{1-\rho} \right]^{\frac{1}{1-\rho}} \]
Two period example

1. Let the agents trade in a complete state-contingent market with information \(\{F_t\}\).

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V_1 = \left[ c_1^{1-\rho} + \beta E \left[ c_2^{1-\gamma} \mid F_1 \right] \right]^{\frac{1-\rho}{1-\gamma}}
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\[F_1^* = F_2.\]

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V_1^* = \left[ c_1^{1-\rho} + \beta E \left[ c_2^{1-\gamma} \mid F_1^* \right] \right]^{\frac{1-\rho}{1-\gamma}} = \left[ c_1^{1-\rho} + c_2^{1-\rho} \right]^{\frac{1}{1-\rho}}
\]

4. Now re-trading can occur: Second round of trading is under different preferences.
Notice that it is crucial that new information arrives as a surprise.

- First round of trading under preference ranking $V_1$. 
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- First round of trading under preference ranking $V_1$.
- Second round of trading under preference ranking $V_1^*$, with $V_1 \neq E[V_1^* \mid \mathcal{F}_1]$.
Notice that it is crucial that new information arrives as a surprise.

- First round of trading under preference ranking $V_1$.
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- Dynamic inconsistency
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If agents in round 1 knew that additional information would arrive before second round:

- First round of trading under preference ranking $E[V_1^* | F_1]$
INTERPRETATION 1: CHANGING THE PREFERENCE STRUCTURE

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- First round of trading under preference ranking $V_1$.
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If agents in round 1 knew that additional information would arrive before second round:

- First round of trading under preference ranking $E[V_1^* \mid \mathcal{F}_1]$
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If agents in round 1 knew that additional information would arrive before second round:

- First round of trading under preference ranking $E[V_1^* | \mathcal{F}_1]$
- Second round of trading under preference ranking $V_1^*$
- *Dynamically consistent* $\implies$ no retrading.
· Agents can contract in markets that are complete wrt to $\theta^t \in \mathcal{F}_t$

· Cannot contract on signals $x_t \in \mathcal{F}_\tau$ about future states in periods $\tau > t$. 
Agents can contract in markets that are complete wrt to \( \theta^t \in \mathcal{F}_t \).

Cannot contract on signals \( x_t \in \mathcal{F}_\tau \) about future states in periods \( \tau > t \).

Under **separable preferences**, \( x_t \) contracts are irrelevant ex ante.

- \( x_t \) is irrelevant for time-\( t \) consumption allocation
- Contracting upon \( \theta^\tau \) is sufficient for time-\( \tau \) consumption allocation
Agents can contract in markets that are complete wrt to $\theta^t \in \mathcal{F}_t$.

Cannot contract on signals $x_t \in \mathcal{F}_\tau$ about future states in periods $\tau > t$.

Under **separable preferences**, $x_t$ contracts are irrelevant ex ante.

- $x_t$ is irrelevant for time-$t$ consumption allocation
- Contracting upon $\theta^\tau$ is sufficient for time-$\tau$ consumption allocation

Under **non-separable preferences**, $x_t$ contracts matter.

- Optimal time-$t$ consumption allocation is a function of the whole history
COMPARING BOTH INTERPRETATIONS

Under \textit{separable preferences}

\begin{itemize}
    \item neither of the experiments leads to retrading
    \item ex post changes in information structure are irrelevant
    \item trading on payoff-nonrelevant signals does not occur
\end{itemize}
COMPARING BOTH INTERPRETATIONS

Under **separable preferences**

- neither of the experiments leads to retrading
- ex post changes in information structure are irrelevant
- trading on payoff-nonrelevant signals does not occur

Under **non-separable preferences**

- these two experiments are distinct
- the paper uses the incomplete market interpretation
THEORETICAL QUESTIONS

1. Paper defines risk-aversion dominating preferences, which, \( \forall C \),

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Which preference specifications satisfy this condition (\( \forall C \))?

- Apart from separable preferences?
- E.g., within the class of Epstein–Zin preferences?
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- Apart from separable preferences?
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2. Why cannot we complete the markets to news signals \( x_t \)?

- Agents would want to trade such contracts. What prevents it?
This is a challenging task.

- Many degrees of freedom that are hard to discipline.
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- Many degrees of freedom that are hard to discipline.
- Quantification of ‘news shocks’ (Barsky and Sims (2011), Sims (2012)) that cannot be contracted upon ex ante.

Right now the quantitative model can generate large amount of retrading (volume).

- Proof of concept?
- Complete markets in payoff-relevant states.
- Perfect signal about next period state that is not contractible.

A more serious exercise should look at

- Precision of signals about the future (news shocks)
- Empirical evidence on (non)contractability of these shocks (derivative markets).