Identifying ambiguity shocks in business cycle models using survey data*

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Abstract

We develop a macroeconomic framework with agents facing time-varying concerns for model misspecification. These concerns lead agents to interpret the economy through the lens of a pessimistically biased 'worst-case' model. We use survey data to identify exogenous fluctuations in the worst-case model. In an estimated New-Keynesian business cycle model with frictional labor markets, these ambiguity shocks explain a substantial portion of the variation in labor market quantities.

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1 Introduction

Equilibrium outcomes in the macroeconomy depend on the belief formation mechanism of economic agents. While the rational expectations assumption is in many cases a fruitful benchmark that allows transparent estimation and testing of economic models using time-series data, there is extensive empirical evidence against this assumption. However, if we are to dispense with rational expectations, we need to replace them with a belief formation framework that preserves structural integrity and testability, and allows us to understand how deviations of agents' subjective beliefs interact with economic dynamics.

In this paper, we provide a tightly specified framework that links agents' decisions and beliefs with observable economic outcomes and survey data. The theoretical foundation of the belief formation mechanism is our extension of the robust preference model of Hansen and Sargent (2001a,b). Agents endowed with robust preferences are concerned that the particular model they view as their 'benchmark' model of the economy may be misspecified. Instead of only using only the benchmark model, they consider a whole set of models that are statistically hard to distinguish from the benchmark model. The concerns for model misspecification lead them to choose the model from this set that delivers the lowest utility. This 'worst-case' model is then the basis for their decisions, akin the utility-minimizing prior in the multiple prior framework of Gilboa and Schmeidler (1989) and Epstein and Schneider (2003). The robust preference framework thus delivers a specific form of ambiguity aversion.

We extend this robust preference framework to allow the agents to be exposed to shocks to their ambiguity aversion. The time-variation in ambiguity aversion induces fluctuations in agents' worst-case beliefs and endogeously affects equilibrium dynamics. While our extension delivers a more flexible specification of the time-variation in the worst-case model, it still tightly restricts the beliefs across alternative states in a given period. Agents fear outcomes with adverse utility consequences and overweigh their probabilities in a specific way.

In order to identify the variation in the worst-case model empirically, we assume that agents' forecasts in the survey data are based on their worst-case model. Our theoretical model yields directly testable predictions about the comovement of these forecasts under the worst-case model. We show that household forecasts for key macroeconomic variables in the University of Michigan Surveys of Consumers are indeed significantly pessimistically biased, with a discernible business cycle component. We start by estimating a vector-autoregression (VAR) that embeds household survey data, explicitly restricting the belief distortion (or wedge) between the worst-case model and the data-generating probability measure. A common component of these belief distortions in different survey answers identifies a latent factor that captures the time-variation in the worst-case model, and its impact on observable macroeconomic quantities.

We then combine the robust preference framework and the survey data in a dynamic stochastic general equilibrium model with frictional labor markets, sticky prices and a monetary authority that follows an interest rate rule. We estimate this model using Bayesian methods and study the quantitative role of the ambiguity shocks in the dynamics of the labor market, idenfitication of the monetary policy rule and the comovement of macroeconomic variables.

The results from the reduced-form and structural models show a common pattern. The worst-case belief is identified from the common variation of the biases in survey answers, and it explains a substantial amount of variation in these biases, in particular in the households' forecasts of unemployment and GDP growth. Ambiguity-averse households interpret high unemployment and low GDP growth states as particularly adverse to their utility, and overweight worst-case probabilities of those states substantially.

An adverse ambiguity shock also has significant contractionary effects, propagated particularly strongly through the labor market. In the labor market with search and matching, creation of new matches and hiring depend on the assessment of the future surpluses generated in a new match. An increase in ambiguity leads to a more pessimistic evaluation of future surpluses and therefore to lower match creation, which increases unemployment and decreases output.

The paper contributes to the growing literature that quantitatively assesses the role of ambiguity aversion in the macroeconomy, building on alternative decision-theoretical foundations by Gilboa and Schmeidler (1989), Epstein and Schneider (2003), Klibanoff et al. (2005, 2009), Hansen and Sargent (2001a,b), Strzalecki (2011) and others. Applications to macroeconomic models include Cagetti et al. (2002) and Bidder and Smith (2012). For a survey of applications in finance, see Epstein and Schneider (2010).

Perhaps the closest to our paper is the work by Ilut and Schneider (2014) and Bianchi et al. (2014) who utilize the recursive multiple-prior preferences of Epstein and Schneider (2003). The first crucial difference lies in the fact that the multiple-prior framework does not impose tight a priori restrictions on the relative distortions of individual shocks under the worst-case model, and thus introduces a heavier burden on identification through observable data. We rely much more strongly on the restrictions on shock distortions implied by the robust preference framework. Second, we use data on cross-sectional average distortions measured in household survey answers, for which our theory has direct quantitative predictions, as a source of identification of the ambiguity shocks. Ilut and Schneider (2014) instead use the forecast *dispersion* as a proxy for confidence and show an empirically plausible relation of this measure to the notion of ambiguity aversion. Despite the differences, both these approaches should be viewed as complementary.

2 Survey expectations

We analyze the data on households' expectations from the University of Michigan Surveys of Consumers (Michigan Survey). These surveys collect answers to questions about the households' own economic situation as well as their forecasts about the future state of the economy. Specifically, we focus on the forecasts of future inflation, unemployment rate and the expected index of consumer sentiment, which we use as a proxy for GDP growth.

We are interested in deviations in these survey answers from rational expectations forecasts. This necessarily requires taking a stand on how to determine the probability measure that generates

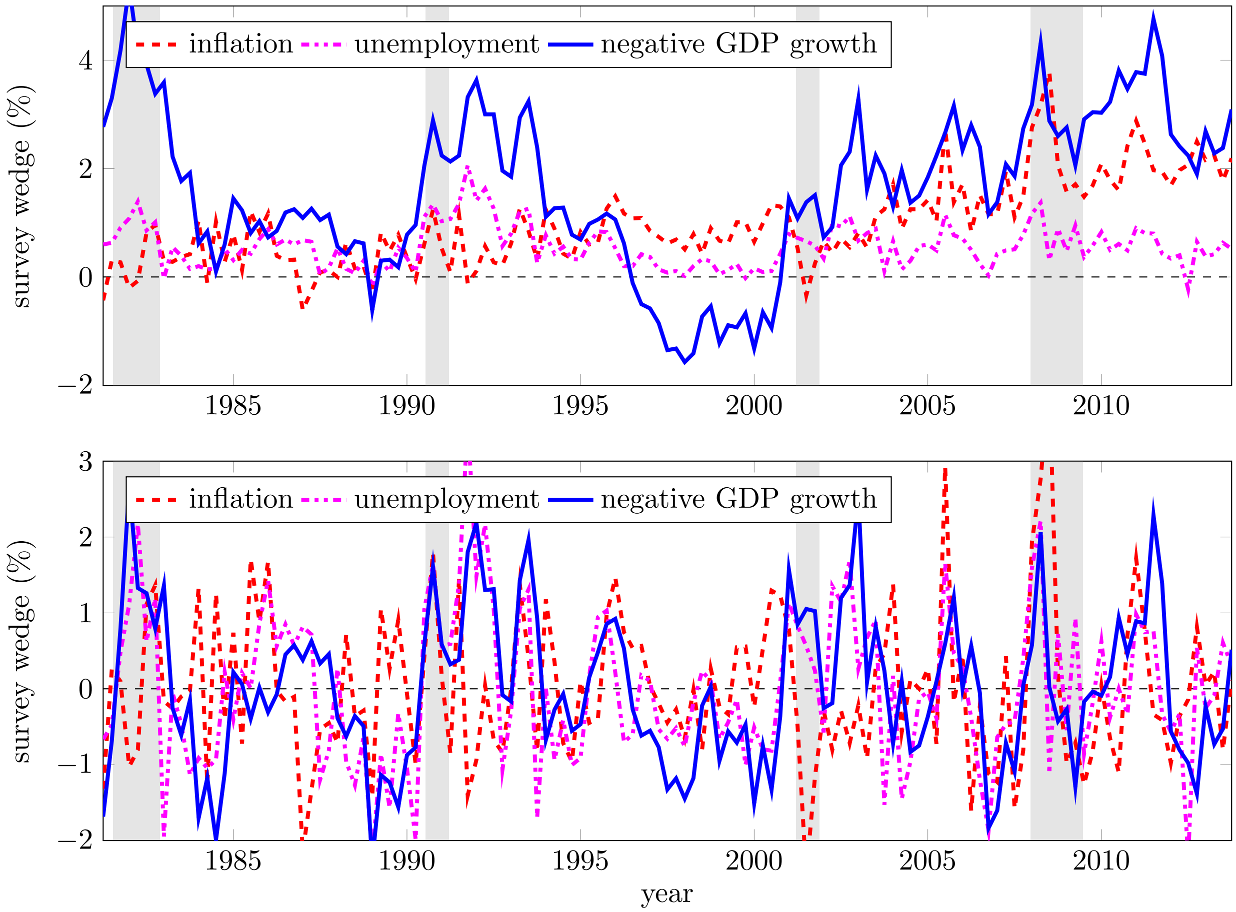

Figure 1: Difference in survey expectations between the Michigan Survey and Survey of Professional Forecasters. Top panel original data, bottom panel HP-filtered and standardized. GDP growth for the Michigan Survey proxied by the expected consumer confidence index. Details on the construction of the data series are in Appendix C. NBER recessions shaded.

the data. Here, we assume that the Survey of Professional Forecasters provides unbiased estimates for the variables we study. In Sections 3 and 5, we also contrast these household survey answers with predictions obtained from VAR and structural models.

Figure 1 shows the differences in survey expectations between the Michigan Survey and the Survey of Professional Forecasters for inflation, unemployment and GDP growth. The survey expectations are mean one-year ahead expectations in the survey samples. The Michigan Survey does not contain a question about GDP growth, and we therefore proxy it by projecting GDP growth on the survey answer on expected consumer sentiment. We detail the construction of the time series in Appendix C.

The top panel of Figure 1 reveals that households' expectations are systematically pessimistically biased — relative to professional forecasters, households overpredict future unemployment and inflation, and underpredict GDP growth (with the exception of the boom period during the late 1990s). Moreover, despite a substantial amount of noise, the three time series for the belief

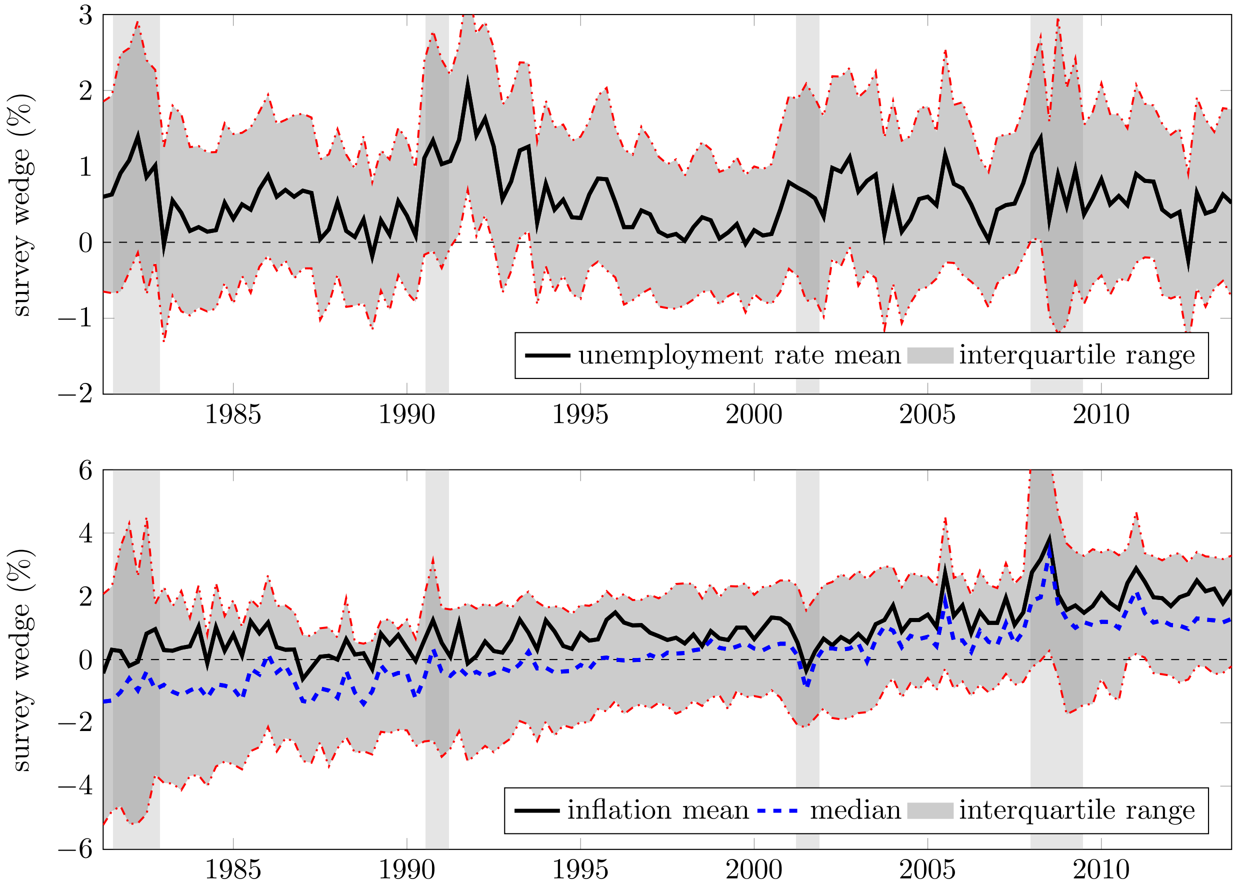

Figure 2: Dispersion in survey expectations in the Michigan Survey. The graphs show different quantiles of the distribution of responses in the Michigan survey, net of the mean response in the Survey of Professional Forecasters. The top panel shows the unemployment responses, bottom panel the inflation responses. Details on the construction of the data series are in Appendix C. NBER recessions shaded.

wedges have a common business cycle component and are statistically significantly correlated. The correlation coefficient for the unemployment and negative GDP growth wedges is 0.52, while the correlation between the inflation and negative GDP growth wedges is 0.31, both with a standard error of 0.07. The comovement over the business cycle can be visually confirmed in the bottom panel of Figure 1 that plots HP-filtered and standardized data.

Our theoretical framework formalizes the notion of pessimistic belief distortions through the structure of the robust preference model. The common component of the three belief wedges from Figure 1 identifies the fluctuations in the worst-case model of economic agents. We embed the belief distortions in a representative agent framework, which provides a justification for using average forecasts as a measure of subjective expectations in the model.

Mankiw et al. (2003), Bachmann et al. (2012) and others use measures of cross-sectional forecast dispersion as a proxy for economic uncertainty. This proxy is typically based on the presumption that a higher dispersion is indicative of more difficulty in estimating the forecast distribution, and therefore implies more ambiguity. Ilut and Schneider (2014) apply the same logic to use the dispersion in the SPF forecasts as a proxy of household confidence in the forecasting model.

We plot the dispersion data from the Michigan survey for the unemployment rate and inflation rate forecasts in Figure 2 for comparison. For the inflation data, we have information on the quantiles of the cross-sectional distribution. For the unemployment rate forecast, we fit a sequence of normal distributions to categorical answers using the same method as in Mankiw et al. (2003), see Appendix C for details.

There is indeed a substantial cross-sectional dispersion in the survey answers across individual households. However, the interquartile range appears to be stable (except for the inflation answers from early 1980s), and in the case of the unemployment answer, also visibly comoves with the business cycle.

While it may be appealing to use cross-sectional dispersion in forecasts as a proxy for the ambiguity concerns of each individual household, our theory does not provide such a direct link. We want to keep ambiguity concerns separate from the notion *disagreement* in forecasts across households. The model we develop in this paper is based on a representative agent framework that does not feature heterogeneity in individual forecasts, and therefore yields no predictions about forecast dispersion measures. However, it is possible to extend the framework by introducing heterogeneity in agents' concerns for uncertainty. Agents with differing degrees of ambiguity aversion deduce alternative worst-case models from observable data, which then generates dispersion in forecasts in the model. While conceptually interesting, this extension is beyond the scope of this paper.

3 A one-factor model of distorted beliefs

We want to formalize the empirical facts that we established in the previous section. We start with a statistical model that describes the joint dynamics of macroeconomic variables and households' expectations. In this model, households' expectations are allowed to differ from the expectations implied by the distribution of the data-generating process. The underlying idea is to extract a common component in the variation of the belief wedges, and study its impact on the dynamics of the macroeconomic variables.

While we specify a flexible, reduced-form specification for the dynamics of observable variables, we impose tight restrictions on the households' expectations. These restrictions reflect those implied by our structural model of robust preferences that we introduce in Section 4.

We specify a $(k-1) \times 1$ vector of observable economic variables y_t and an unobservable scalar latent process f_t. In particular, consider the model

$$\begin{aligned} y_{t+1} &= \psi_y y_t + \psi_{yf} f_{t+1} + \psi_{yw} w^y_{t+1} \\ f_{t+1} &= \rho_f f_t + \sigma_f w^f_{t+1} \end{aligned}$$

where $w'_{t+1} = \left(\left(w^y_{t+1} \right)', w^f_{t+1} \right) \sim N\left(0, I_k\right)$ is a $k \times 1$ vector of normally distributed iid shocks. We can rewrite these equations, expressing the joint process $(y_t', f_t)'$ as follows:

$$\begin{pmatrix} y_{t+1} \\ f_{t+1} \end{pmatrix} = \begin{pmatrix} \psi_y & \psi_{yf}\rho_f \\ 0 & \rho_f \end{pmatrix} \begin{pmatrix} y_t \\ f_t \end{pmatrix} + \begin{pmatrix} \psi_{yw} & \psi_{yf}\sigma_f \\ 0 & \sigma_f \end{pmatrix} \begin{pmatrix} w_{t+1}^y \\ w_{t+1}^f \end{pmatrix}. \tag{1}$$

This process generates a filtered probability space $(\Omega, \{\mathcal{F}_t\}_{t=0}^{\infty}, P)$ where P is the objective, data-generating probability measure. The factor f_t is exogenous to the dynamics of macroeconomic variables and will serve as a source of common variation in the dynamics of the macroeconomy and households' belief wedges.

Households' expectations are represented by a subjective probability measure $\widetilde{P}$ that can differ from P. In Section 4, we derive a formal structural model for $\widetilde{P}$. Here, we focus on imposing restrictions on $\widetilde{P}$ that are consistent with the structural model and that allow us to identify $\widetilde{P}$ using household survey data.

Let z_t be a subset of observable variables y_t for which survey data are available. We define the τ-period belief wedge $\Delta_t^{(\tau)}$ as the difference between the τ-period forecasts under the beliefs of the households and under objective expectations:

$$\Delta_t^{(\tau)} \doteq \widetilde{E}_t z_{t+\tau} - E_t z_{t+\tau}$$

where $\widetilde{E}_t z_{t+\tau}$ is the time-t expectation of $z_{t+\tau}$ under the subjective probability measure of the households. In addition we define the τ-period average belief wedge $\overline{\Delta}_t^{(\tau)}$ as the the average difference in forecasts under the beliefs of the households and under objective expectations:

$$\overline{\Delta}_t^{(\tau)} \doteq \frac{1}{\tau} \sum_{s=1}^{\tau} \Delta_t^{(s)}$$

We impose that the dynamics of belief wedges $\Delta_t^{(\tau)}$ and $\overline{\Delta}_t^{(\tau)}$ can be summarized using the scalar factor

$$\theta_t = (F_y, F_f) \begin{pmatrix} y_t \\ f_t \end{pmatrix}. \tag{2}$$

Individual one-period forecasts of the innovation means under the households' expectations are then represented by a vector of factor loadings H:

$$\widetilde{E}_t \left[w_{t+1} \right] = H\theta_t. \tag{3}$$

Applying the law of iterated expectations, belief wedges for the τ-period forecasts can be written as

$$\Delta_t^{(\tau)} = G_x^{(\tau)} x_t + G_0^{(\tau)}$$

where the coefficients $G_x^{(\tau)}$ and $G_0^{(\tau)}$ are derived in Appendix A. The model (2)–(3) thus implies a one-factor structure of belief wedges where θ_t captures the common comovement in the belief wedges. In this reduced form model, we interpret θ_t as the time-varying measure of pessimism among the households reflected in the survey data that impacts the dynamics of macroeconomic variables. In Section 5, this one-factor structure is derived from the decision problem of the household endowed with robust preferences, where θ_t reflects the time-variation in households' ambiguity concerns.

3.1 Data and estimation

Data on macroeconomic variables are obtained from the Federal Reserve Bank of St. Louis database (FRED), at quarterly frequency. The vector y_t includes real log GDP growth, log inflation, the unemployment rate, and the Federal Funds rate. We include three belief wedges from Figure 1 in the vector $\overline{\Delta}_t^{(4)}$, constructed as 4-quarter ahead average belief wedges between the Michigan Survey and SPF forecasts for log GDP growth, the unemployment rate and log inflation. Appendix C provides details on the construction of the data, presented in Section 2. The data for y_t covers the period 1951Q2–2013Q3. The belief wedges for the unemployment rate, GDP growth and inflation cover the periods 1977Q4–2013Q3, 1968Q4–2013Q3 and 1981Q2–2013Q3 respectively.

In order to keep the estimation and interpretation of the model transparent, we restrict the dynamics of beliefs and set $F_y = 0$, thereby setting $\theta_t = f_t$. This implies that fluctuations in the belief wedges are driven purely by the belief factor f_t, and not directly by the dynamics of endogenous macroeconomic variables y_t. In addition, we normalize $F_f = 1$ and set the element of H corresponding to the GDP growth shock to be -1 in order to identify the sign and scale of θ_t.

The shock exposure matrix ψ_{yw} is only identified as the covariance matrix $\psi_{yw}\psi_{yw}'$. For the purpose of estimation, we shall impose a recursive identification scheme for ψ_{yw}. However, ψ_{yw} only appears as $\psi_{yw}\psi_{yw}'$ in the formulas for the belief wedges. Therefore, given an estimate of $\psi_{yw}\psi_{yw}'$, the identification of ψ_{yw} does not play a role in the estimation of the factor shocks w_t^f.

More specifically, we estimate the model (1) together with a vector of observation equations for the wedges

$$\overline{\Delta}_{t+1}^{(4)} = \psi_{\Delta f} f_{t+1} + \sigma_\Delta \varepsilon_{t+1}^\Delta$$

where σ_Δ is diagonal and $\varepsilon_{t+1}^\Delta \sim N(0, I)$ is a vector of normally distributed iid measurement errors. y_t and $\overline{\Delta}_t^{(4)}$ are demeaned. We seek estimates for the parameters

$$\{\psi_y, \psi_{yf}, \psi_{yw}, \rho_f, \sigma_f, H, \sigma_\Delta\}$$

and the belief factor $\theta_t = f_t$. Appendix A solves for $\psi_{\Delta f}$ from the above parameters.

We estimate the model using Bayesian methods. Further details, including the imposed priors and estimated posteriors are summarized in Appendix D.

3.2 Results

A variance decomposition at the modal parameter estimate, summarized in Table 4 in Appendix D, reveals that the factor shock explains 55.5%, 43.9%, and 7.5% of the variation in the output wedge,

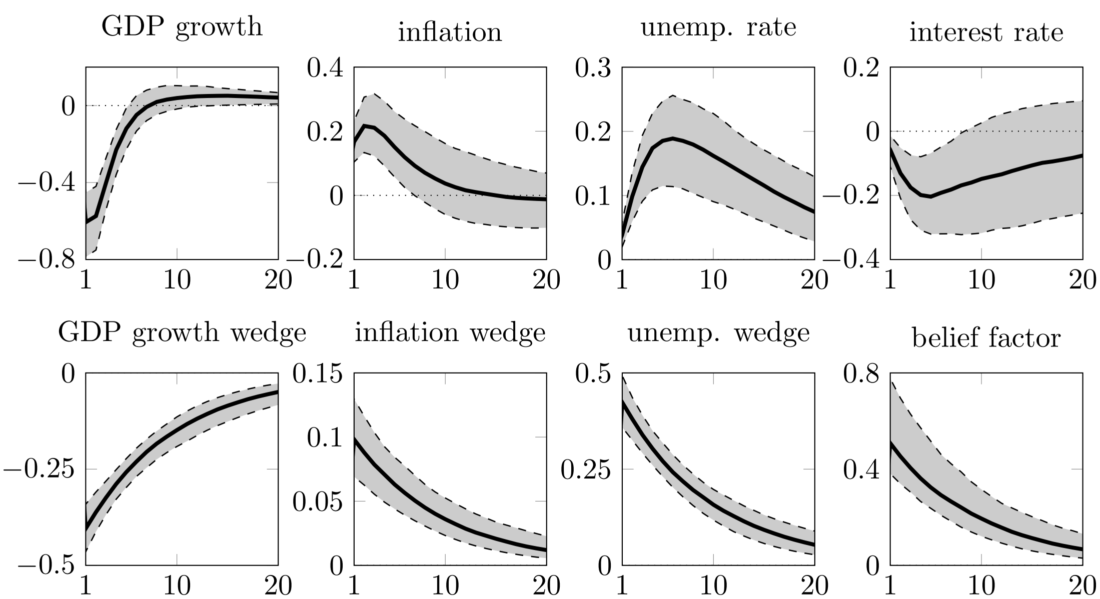

Figure 3: Bayesian impulse response functions in the one-factor model to the belief shock w^f. The solid lines indicate median estimates, while the dashed line correspond to 10th and 90th percentile error bands. Output growth, inflation, and interest rate are annualized and in percentage deviations. The unemployment rate is in percentage points. The output wedge and inflation wedge are scaled to correspond to the belief wedges of annualized output growth and annualized inflation. The horizontal axis is in quarters.

unemployment wedge, and inflation wedge respectively. These results confirm the strong correlation between the belief wedges that concern real quantities. The fact that a sizeable fraction of variation in the wedges is explained by θ_t supports the single factor model. Moreover, the posterior estimates shown in Table 4 in Appendix D reveal a very tightly identified persistence ψ_f of this factor with posterior mean of 0.9 at the quarterly frequency.

Figure 3 plots the impulse response functions of y_t and $\overline{\Delta}_t^{(4)}$ to a positive one standard deviation shock w_t^f to $\theta_t = f_t$, with factor response plotted in the bottom right panel. We find a negative impulse response for the output belief wedge and a positive impulse response for the unemployment and inflation wedges in response to a positive shock to θ_t. An increase in θ_t leads household forecasts for GDP growth to be biased further downward relative to the SPF forecasts, while the biases in the household forecasts for unemployment and inflation increase relative to the SPF forecasts. The impulse responses of the belief wedges are consistent with the correlations and average signs of the wedges described in Section 2.

These results are consistent with the interpretation of θ_t as a time-varying measure of the level of pessimism among households. From the perspective of the robust preference model that we develop in the next section, households are concerned about a future path that exhibits low GDP growth, a high unemployment rate and high inflation. An increase in θ_t makes these concerns stronger, biasing households' beliefs more strongly in this direction.

The belief shock also has real effects. In response to a positive shock to θ_t, GDP growth falls and unemployment rises. The impulse response for inflation is positive for the first year and close to zero subsequently. Interest rates exhibit a negative median response with large error bands that contain zero. At the modal parameter estimate, θ_t explains 7.4%, 25.4%, 3.2% and 3.1% of the movements in GDP growth, unemployment, inflation and interest rates, respectively. Our estimates therefore suggest that a rise in pessimism has contractionary effects, and we emphasize the especially large adverse response of unemployment.

In Section 5, we develop and estimate a structural macroeconomic model with a frictional labor market and ambiguity averse agents and revisit these empirical findings. In line with the results from the factor model, the ambiguity shock in the structural model generates nontrivial recessionary responses, with a particularly pronounced response in the labor market.

4 Robust preferences

Motivated by the empirical results from Sections 2 and 3, we now introduce a preference model that generates endogenous deviations of agents' beliefs from the data-generating probability measure. This model extends the robust preference framework of Hansen and Sargent (2001a,b) to allow for more flexible form of belief distortions, similar to Hansen and Sargent (2015). The flexibility allows for time-variation in the degree of agents' pessimism over time, which we identify from survey data, while tightly restricting the structure of pessimistic distortions across individual states, linking them to agents' preferences.

We consider a class of Markov models for the equilibrium dynamics

$$x_{t+1} = \psi\left(x_t, w_{t+1}\right) \tag{4}$$

where x_t is an $n \times 1$ vector of stationary economic variables and $w_{t+1} \sim N\left(0_k, I_{k\times k}\right)$ an iid vector of normally distributed shocks under the data-generating probability measure P. Agents are endowed with a version of robust preferences that satisfy the continuation value recursion

$$V_t = \min_{\substack{m_{t+1}>0 \\ E_t[m_{t+1}]=1}} u\left(x_t\right) + \beta E_t\left[m_{t+1} V_{t+1}\right] + \frac{\beta}{\theta_t} E_t\left[m_{t+1} \log m_{t+1}\right] \tag{5}$$

with period utility $u\left(x_t\right)$. These preferences have been formulated by Hansen and Sargent (2001a,b) as a way of introducing concerns for model misspecification on the side of the agents. The agent treats model (4) as an approximating or benchmark model and considers potential stochastic deviations from this model, represented by the strictly positive, mean-one random variable m_{t+1}. The minimization problem in (5) captures the search for a 'worst-case' model that serves as a basis for the agent's decisions. The models that are considered by the agent are difficult to distinguish statistically from the benchmark model, and the degree of statistical similarity is controlled by the entropy penalty $E_t\left[m_{t+1} \log m_{t+1}\right]$, scaled by the penalty parameter θ_t. More pronounced statistical deviations that are easier to detect are represented by random variables m_{t+1} with a large dispersion that yields a large entropy.

The preferences considered by Hansen and Sargent (2001a,b) impose a constant parameter $\theta > 0$. As $\theta \searrow 0$, the penalty for deviating from the benchmark model becomes more severe, and the resulting preferences are closer to a utility-maximizing household with rational expectations.

We are interested in a specification that permits more flexibility in the set of models that the households views as plausible. In particular, we envision the time-varying model

$$\theta_t = \overline{\theta} x_t. \tag{6}$$

where $\overline{\theta}$ is a $1 \times n$ vector of parameters. It is well-known that the worst-case model distortion relative to the benchmark model given by the solution of (5) satisfies

$$m_{t+1} = \frac{\exp\left(-\theta_t V_{t+1}\right)}{E_t\left[\exp\left(-\theta_t V_{t+1}\right)\right]}. \tag{7}$$

The variation in θ_t thus implies a time-varying model for the worst-case distortion. The chained sequence of random variables m_{t+1} specifies a strictly positive martingale M recursively as $M_{t+1} = m_{t+1} M_t$ with $M_0 = 1$ that defines a probability measure $\widetilde{P}$ with conditional expectations

$$\widetilde{E}_t\left[x_{t+1}\right] \doteq E_t\left[m_{t+1} x_{t+1}\right].$$

Consequently, the wedge between the one-period forecasts of x_{t+1} under the worst-case and benchmark models is given by

$$\Delta_t \doteq \widetilde{E}_t\left[x_{t+1}\right] - E_t\left[x_{t+1}\right]. \tag{8}$$

Notice that the distortion (7) implies a large value of m_{t+1} for low realizations of the continuation value V_{t+1}. The worst-case model, represented by the probability measure $\widetilde{P}$, thus overweighs adverse states as ranked by the preferences of the agent. In this way, the preference model implies tightly restricted endogenous pessimism on the side of the agents, generated by concerns for model misspecification. The degree of pessimism is controlled by the evolution of θ_t.

4.1 A linear approximation

We are interested in deriving a tractable approximation of the equilibrium dynamics (4) and the worst-case biases Δ_t in (8). Assuming that the function $\psi(x, w)$ is sufficiently smooth, we combine the series expansion method of Holmes (1995) and Lombardo (2010) with an extension of the worst-case model approximation used in Borovička and Hansen (2013, 2014). The method, outlined in detail in Appendix B, approximates the dynamics in the neighborhood of the deterministic steady state $\bar{x}$ that is given by the solution to $\bar{x} = \psi(\bar{x}, 0)$. The dynamics of the state vector x_t can be approximated as

$$x_t \approx \bar{x} + \mathsf{q} x_{1t}$$

where q is a perturbation parameter. The law of motion for the 'first-derivative' process x_{1t} that represents the local dynamics in the neighborhood of the steady state can be derived from the linear approximation of (4):

$$x_{1t+1} = \psi_q + \psi_x x_{1t} + \psi_w w_{t+1} \tag{9}$$

where ψ_q, ψ_x and ψ_w are conforming coefficient matrices. Similarly, we can construct a linear approximation of the continuation value (5) where the deviation of the continuation value from its steady state satisfies

$$V_{1t} = V_x x_{1t} + V_q.$$

We show in Appendix B that under the household's worst-case model $\widetilde{P}$, the innovations w_{t+1} are distributed as

$$w_{t+1} \sim N\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right)\left(V_x \psi_w\right)', I_{k\times k}\right).$$

Instead of facing a vector of zero-mean shocks w_{t+1}, the agent perceives these shocks as having a time-varying drift. The time-variation is determined by a linear approximation to θ_t from equation (6), given by $\overline{\theta}\left(\bar{x} + x_{1t}\right)$. The relative magnitudes of the distortions of individual shocks are given by the sensitivity of the continuation value to the dynamics of the state vector, V_x, and the loadings of the state vector on individual shocks, ψ_w. The agent perceives larger distortions during periods when θ_t is large, and distorts relative more the shocks which impact the continuation value more strongly.

Consequently, the dynamics of the model (9) under the agents' worst-case beliefs satisfy

$$\begin{aligned} x_{1t+1} &= \left[\psi_q - \psi_w \psi_w' V_x' \overline{\theta} \bar{x}\right] + \left[\psi_x - \psi_w \psi_w' V_x' \overline{\theta}\right] x_{1t} + \psi_w \widetilde{w}_{t+1} \\ &= \widetilde{\psi}_q + \widetilde{\psi}_x x_{1t} + \psi_w \widetilde{w}_{t+1}. \end{aligned} \tag{10}$$

The worst-case model alters both the conditional mean and the persistence of economic shocks. Moreover, variables that tend to move ambiguity and the continuation value in opposite directions tend to exhibit a higher persistence under the worst-case model.[1]

4.2 Worst-case model and survey responses

In Section 3, we estimated a one-factor model of biases embedded in survey responses on household expectations of future economic variables. The extracted belief biases indicated that households substantially overweigh states which can be viewed as adverse, and that these biases exhibit a non-negligible variation over the business cycle. We also extracted a one-factor structure underlying these belief biases.

The preference framework introduced in this section implies that agents' actions are based on forecasts under the worst-case probability distribution $\widetilde{P}$. We connect the empirical observations on survey responses and the theoretical predictions on decisions under robust preferences and

[1]This statement is precisely correct in the scalar case, when $\psi_x^2 V_x \overline{\theta} < 0$.

hypothesize that surveyed households provide answers regarding economic forecasts having in mind the same probability distribution $\widetilde{P}$.

Using the survey data and the rational forecasts from the linearized model (9), we identify the belief wedges (8) as

$$\Delta_t^{(1)} = \psi_w \widetilde{E}_t \left[w_{t+1} \right] = -\overline{\theta} \left(\bar{x} + x_{1t} \right) \left(\psi_w \psi_w' \right) V_x'. \tag{11}$$

The one-factor structure in survey answers is driven by the time-variation in $\overline{\theta} \left(\bar{x} + x_{1t} \right)$, with the constant vector of loadings $- \left(\psi_w \psi_w' \right) V_x'$.

Observe that this specification of belief wedges is a restricted case of the reduced-form model (1)–(3). In the notation from Appendix A, we have

$$F = \overline{\theta}, \qquad H = - \left(\psi_w \psi_w' \right) V_x', \qquad \overline{H} = -\overline{\theta} \bar{x} \left(\psi_w \psi_w' \right) V_x'.$$

The terms $\overline{\theta}$, ψ_w, V_x are functions of structural parameters in the model. Belief wedges for longer-horizon forecasts can then be computed using formulas from Appendix A.

4.3 Dealing with non-stationarities

For the purpose of applying the expansion method, we assumed that the state vector x_t is stationary. Our framework can, however, deal with deterministic or stochastic trends featured in macroeconomic models. Specifically, let us assume that there exists a vector-valued stochastic process z_t such that the dynamics of x_t can be written as

$$\begin{aligned} x_t &= \widehat{x}_t + z_t \\ z_{t+1} - z_t &= \phi \left(\widehat{x}_t, w_{t+1} \right) \end{aligned} \tag{12}$$

where $\widehat{x}_t$ is a stationary vector Markov process that replaces dynamics (4):

$$\widehat{x}_{t+1} = \psi \left(\widehat{x}_t, w_{t+1} \right)$$

The process z_t thus has stationary increments and x_t and z_t are cointegrated, element by element. A typical example of an element in z_t is a productivity process with a permanent component. Once we solve for the stationary dynamics of $\widehat{x}_t$, we can obtain the dynamics of x_t in a straightforward way using (12).

In order to compute the stationary version of the continuation value recursion and the appropriate worst-case distortions, consider as an example

$$u \left(x_t \right) = \log C_t = \log \left[\widehat{C}_t \exp \left(z_t \right) \right] = \log \widehat{C}_t + z_t \tag{13}$$

where C_t is agent's consumption process and $\widehat{C}_t = \widehat{C} \left(\widehat{x}_t \right)$ is its stationary rescaling. We show in

Appendix B.6 that in this case, the continuation value can be written as

$$V_t = \widehat{V}\left(\widehat{x}_t\right) + \frac{1}{1-\beta} z_t \tag{14}$$

and the worst-case model distortion is given by

$$m_{t+1} = \frac{\exp\left(-\theta_t \left(\widehat{V}\left(\widehat{x}_{t+1}\right) + (1-\beta)^{-1}\phi\left(\widehat{x}_t, w_{t+1}\right)\right)\right)}{E_t\left[\exp\left(-\theta_t \left(\widehat{V}\left(\widehat{x}_{t+1}\right) + (1-\beta)^{-1}\phi\left(\widehat{x}_t, w_{t+1}\right)\right)\right)\right]}.$$

This type of belief distortion has stationary increments m_{t+1} and can be dealth with by applying the first-order series expansion to the functions $\widehat{V}\left(\widehat{x}_{t+1}\right)$ and $\phi\left(\widehat{x}_t, w_{t+1}\right)$ as above. Consequently, the worst-case distribution of the shock vector is given by

$$w_{t+1} \sim N\left(-\overline{\theta}\left(\bar{x} + \widehat{x}_{1t}\right)\left(V_x\psi_w + (1-\beta)^{-1}\phi_w\right)', I_{k\times k}\right).$$

The distortions thus inherit the contribution of the increment $(1-\beta)^{-1}\phi_w$ of the non-stationary process z_t to the dynamics of the continuation value. The worst-case dynamics (10) and the belief wedges (11) are modified accordingly. Specifically, we can compute the multiperiod belief wedges $\Delta_t^{(\tau)}$ using the recursive calculations outlined in Appendix A, imposing

$$\begin{aligned}
F &= \overline{\theta} \\
H &= -\psi_w\left(V_x\psi_w + (1-\beta)^{-1}\phi_w\right)' \\
\overline{H} &= -\left(\overline{\theta}\bar{x}\right)\psi_w\left(V_x\psi_w + (1-\beta)^{-1}\phi_w\right)'.
\end{aligned}$$

5 A structural business cycle model

In this section we introduce the robust preference framework from Section 4 into a dynamic stochastic general equilibrium model of the macroeconomy. We use a version of the New-Keynesian framework with a frictional labor market introduced in Ravenna and Walsh (2008) and Christiano et al. (2015). The frictional labor market with search and matching features and nominal rigidities provides a well-defined notion of unemployment and inflation which directly map to the survey questions.

In Section 3, we used a reduced form VAR specification with a one-factor structure in beliefs to extract a latent component that accounts for the co-movement between the belief wedges in the data and impacts macroeconomic dynamics. In this section, our strategy is to use an estimated version of the structural model to quantify the role and channels through which ambiguity shocks affect the dynamics of realized outcomes and associated belief wedges.

Our choice of the model with a frictional labor market is directly influenced by the empirical findings from Section 3, where the belief shock in the reduced-form model had a particularly significant impact on the unemployment rate. In the search and matching environment, incentives of workers and firms to search for jobs and post job vacancies depend on their forecasts about the present value of a potential match. Ambiguity shocks impact this present value by overweighting the probability of states with low continuation values for the households, which are correlated with low values of the worker-firm matches.

5.1 Model

5.1.1 Representative household

The representative household is endowed with robust preferences given by the recursion (5) with time preference coefficient β and period utility over aggregate consumption C_t,

$$u(x_t) = \log(C_t - bC_{t-1})$$

where b determines the degree of habit formation. In line with our factor model specification from Section 3, we assume that the stochastic process for the robust concerns is given by $\theta_t = \overline{\theta} x_t \doteq f_t$ where f_t follows an AR(1) process

$$f_{t+1} = (1-\rho_f)\,\overline{f} + \rho_f f_t + \sigma_f w_{t+1}^f. \tag{15}$$

The worst-case belief of the household is

$$m_{t+1} = \frac{\exp(-\theta_t V_{t+1})}{E_t[\exp(-\theta_t V_{t+1})]}.$$

The magnitude of the belief distortion is determined by fluctuations in θ_t specified exogenously in (15). However, the equilibrium dynamics in the model endogenously determines which states yields low continuation values V_{t+1} and are therefore evaluated as adverse by the household. These states are then perceived as more likely under the worst-case model. Naturally, the dynamics of the worst-case belief will then endogenously depend on other sources of shocks introduced into the model, which we describe in Section 5.1.4.

The household faces the budget constraint

$$P_t C_t + P_{I,t} I_t + B_{t+1} \leq \left(R_{K,t} u_t^K - a_u\left(u_t^K\right) P_{I,t}\right) K_t + (1-l_t)\, P_t D_t + W_t l_t + R_{t-1} B_t - T_t.$$

P_t is the price of consumption goods and $P_{I,t}$ is the price of investment goods. B_{t+1} denotes the one-period risk-free bonds purchased in period t with return R_t. I_t is the quantity of investment goods. $R_{K,t}$ is the rental rate of capital services, K_t is the household's stock of capital at the start of period t, and $a_u\left(u_t^K\right)$ is the cost of the capital utilization rate u_t^K. Finally, T_t denotes lump sum taxes net of profits.

The household's capital stock evolves according to

$$K_{t+1} = (1-\delta_K) K_t + \left(1 - a_I \left(\frac{I_t}{I_{t-1}}\right)\right)$$

where $a_I(\cdot)$ is an adjustment cost that is increasing and convex.

5.1.2 Labor market

The household consists of a unit mass of workers who perfectly share consumption risk. Fraction l_t is employed and earns a wage ξ_t. Fraction $1-l_t$ is unemployed and collect unemployment benefits D_t financed through lump sum taxes. At the end of period t, employed workers separate with probability $1-\rho$ and join the pool of unemployed who search for jobs at the beginning of period $t+1$. The total number of searchers at the beginning of period $t+1$ therefore is $1-\rho l_t$ and these searchers face a job finding probability j_{t+1}. The law of motion for employed workers thus is

$$l_{t+1} = \rho l_t + (1-\rho l_t) j_{t+1} = (\rho + \eta_{t+1}) l_t$$

where

$$\eta_{t+1} = \frac{j_{t+1}(1-\rho l_t)}{l_{t-1}}$$

is the hiring rate. The value of an employed worker is

$$W_t = \xi_t + \widetilde{E}_t \left[\frac{S_{t+1}}{S_t}\left((\rho + (1-\rho) j_{t+1}) W_{t+1} + (1-\rho)(1-j_{t+1}) U_{t+1}\right)\right]$$

where S_{t+1}/S_t is the period t stochastic discount factor, ξ_t is the period t wage, and U_{t+1} is the value of being unemployed next period, given by the recursion

$$U_t = D_t + \widetilde{E}_t \left[\frac{S_{t+1}}{S_t}\left(j_{t+1} W_{t+1} + (1-j_{t+1}) U_{t+1}\right)\right].$$

Denote by ϑ_t the real marginal revenue in period t from hiring an additional worker. The value of the worker to a firm is given by the revenue generated in the match net of the wages paid,

$$J_t = \vartheta_t - \xi_t + \rho \widetilde{E}_t \left[\frac{S_{t+1}}{S_t} J_{t+1}\right].$$

We assume free entry of firms, so that in equilibrium,

$$Q_t (J_t - \kappa_t) = s_t$$

where Q_t is the probability of filling a vacancy, κ_t is the fixed cost of hiring a worker, and s_t is the cost of posting a vacancy. The expectations operators in the recursions indicate that both the workers and the firms evaluate the distribution of future quantities under the worst-case measure

$\widetilde{P}$.

The expectations operators in the recursions indicate that both the workers and the firms evaluate the distribution of future values of W_t, U_t and J_t under the worst-case measure $\widetilde{P}$. As long as the values of the match to the worker and to the firm, and the value of unemployment correlate with the continuation value of the household V_t, they will be impacted by the fluctuations in ambiguity concerns of the representative household. This is a striking difference relative to the Walrasian spot market where workers are hired only using one-period employment contracts. In such an environment, ambiguity concerns are absent from the labor market decisions, since there is no uncertainty about economic conditions prevailing in the given period.

What remains to be determined is the split of the surplus from a match between the firm's surplus, J_t , and the worker's surplus, $W_t - U_t$. As in Hall and Milgrom (2008) and Christiano et al. (2015), we adopt the alternating offer bargaining protocol of Rubinstein (1982) and Binmore et al. (1986). The details of the bargaining protocol are outlined in Appendix E where we show that the outcome of this bargaining mechanism is

$$J_t = \beta_1 (W_t - U_t) - \beta_2 \gamma_t + \beta_3 (\vartheta_t - D_t)$$

with parameters β_i, $i = 1, 2, 3$ that depend on the parameters of the bargaining problem. Notice that when $\beta_2 = \beta_3 = 0$, we obtain the Nash bargaining solution with workers' share $(1 + \beta_1)^{-1}$. Relative to the Nash bargaining solution, the alternative offer bargaining makes the firms' surplus more procyclical, leading to smoother wages and more procyclical hiring patterns over the business cycle.

5.1.3 Production and market clearing

The frictional labor market is embedded in a New-Keynesian framework with Calvo (1983) price setting. A homogeneous final good Y_t with price P_t is produced in a competitive market using the production technology

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda}} \, di \right]^{\lambda}, \qquad \lambda > 1.$$

where $Y_{i,t}$ are specialized inputs with prices $P_{i,t}$. Final good producers solve the static competitive problem

$$\max_{Y_{i,t}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di,$$

leading to the first-order conditions

$$Y_{i,t} = \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda}{\lambda - 1}} Y_t, \qquad i \in [0, 1] \, .$$

Specialized inputs are produced by monopolist retailers indexed by i, using the production technology

$$Y_{i,t} = k_{i,t}^{\alpha} \left(A_t h_{i,t}\right)^{1-\alpha} - \phi_t$$

where $k_{i,t}$ is the quantity of capital purchased, $h_{i,t}$ is the quantity of intermediate goods, A_t is the neutral technology level, and ϕ_t is a fixed cost of production. The retailer purchases intermediate goods at price P_t^h from a wholesaler in a competitive market. We assume that the retailer must borrow $P_t^h h_{i,t}$ at the nominal interest rate R_t. The loan is repaid at the end of period t after the retailer receives its sales revenues. Finally, the retailer is subject to the sticky price friction, implying that every period he is allowed to reset the price with probability $1-\chi$.

Intermediate goods are produced by wholesalers using a technology that turns one unit of labor into one unit of capital. Therefore the market clearing condition for intermediate goods is

$$\int_0^1 h_{i,t} di = h_t = l_t.$$

Market clearing for services of capital requires

$$\int_0^1 k_{i,t} di = u_t^K K_t.$$

Lastly, we have the aggregate resource constraint

$$C_t + \left(I_t + a_u\left(u_t^K\right) k_t\right)/\Psi_t + \left(s_t/Q_t + \kappa_t\right)\eta_t l_{t-1} + G_t = Y_t$$

where G_t denotes government consumption and $\Psi_t = P_t/P_{I,t}$ denotes the relative price of investment and reflects investment-specific technological progress.

5.1.4 Shock structure and monetary policy

We complete the model by specifying the sources of exogenous variation to the model. We assume that the monetary authority follows the interest rate policy rule

$$\log\left(R_t/\overline{R}\right) = \rho_R \ln\left(R_{t-1}/\overline{R}\right) + \left(1-\rho_R\right)\left[r_\pi \log\left(\pi_t/\overline{\pi}\right) + r_y \log\left(\mathcal{Y}_t/\mathcal{Y}_t^*\right)\right] + \sigma_R w_t^R$$

where w_t^R is a monetary policy shock and

$$\mathcal{Y}_t = C_t + I_t/\Psi_t + G_t$$

denotes real GDP. $\mathcal{Y}_t^*$ is the value of $\mathcal{Y}_t$ along the non-stochastic steady state growth path, scaled by the current level of productivity.

Finally, we prescribe the dynamics of technology shocks. The macroeconomic literature alternatively specifies the technology shocks as either persistent but trend-stationary, or unit-root processes. The disagreement is driven by the difficulty to statistically distinguish these two types of processes in available data samples. It turns out that the magnitude of the fluctuations in the belief wedges strongly support the unit-root specification of the technology process, and issue that we discuss in detail in Section 6.

This leads us to assume that the neutral technology process A_t exhibits iid growth

$$\log\left(A_t/A_{t-1}\right) \doteq \log\left(\mu_{A,t}\right) = \sigma_A w_t^A$$

while the investment-specific technological process Ψ_t has a mean-reverting growth rate

$$\log\left(\Psi_t/\Psi_{t-1}\right) \doteq \log\left(\mu_{\Psi,t}\right) = \rho_\Psi \log\left(\mu_{\Psi,t-1}\right) + \sigma_\Psi w_t^\Psi.$$

The final source of exogenous variation is the ambiguity shock process (15). We assume that all innovations are independent under the data-generating measure P:

$$\left(w_t^R, w_t^A, w_t^\Psi, w_t^f\right)' \overset{iid}{\sim} N\left(0, I\right).$$

As we have seen in Section 4, this property does not carry over to the worst-case model where the distribution of future realizations of the shocks depends on the current level of ambiguity concern θ_t.

5.2 Estimation

We are interested in studying the quantitative role of ambiguity shocks in the joint dynamics of output, unemployment, inflation and interest rates through the lens of the structural model introduced above. The impact of these shocks on the economy is restricted through the structure of the model, and we use survey data as a new source of information to aid identification.

As in the reduced form analysis in Section 3, we use data on the unemployment rate, federal funds rate, inflation rate, inflation wedge, unemployment wedge and output wedge with iid measurement errors on the three wedges.[2] We estimate the model using Bayesian methods. In order to make the estimation tractable and transparent, we calibrate a subset of parameters to values listed in the bottom part of Table 1, and focus our estimation on the parameter vector

$$\left\{\rho_R, r_\pi, r_y, \sigma_R, \sigma_A, \rho_\Psi, \sigma_\Psi, \rho_f, \sigma_f, \sigma_{\Delta,\pi}, \sigma_{\Delta,u}, \sigma_{\Delta,y}\right\},$$

which consists of parameters associated with the monetary policy rule and the underlying shock processes. The last three parameters are the standard deviations on the measurement errors. Our priors for the Taylor rule coefficients and stochastic processes for technology and monetary policy shocks are similar to Christiano et al. (2015).

The first part of Table 1 summarizes the results of our estimation. The posterior distributions are plotted in Appendix E.

[2] The details of the data construction are in Appendix C.

Parameter		Prior D,Mean,Std	Posterior Mean, 90% HPD
Monetary Policy			
ρ_R	Taylor Rule: Smoothing	$\mathcal{B}(0.8, 0.1)$	0.78, (0.76, 0.80)
r_π	Taylor Rule: Inflation	$\mathcal{G}(1.7, 0.1)$	1.61, (1.5, 1.72)
r_y	Taylor Rule: Output	$\mathcal{G}(0.04, 0.05)$	0.02, (0.016, 0.03)
Shock Processes			
$100\sigma_R$	Std. Monetary Policy	$\mathcal{G}(0.1, 1)$	0.24, (0.22, 0.26)
$100\sigma_{\mu_z}$	Std. Neutral Tech. Shock	$\mathcal{G}(0.1, 1)$	0.93, (0.87, 1)
$100\sigma_{\mu_\Psi}$	Std. Invest. Tech Shock	$\mathcal{G}(0.1, 1)$	2.91, (2.61, 3.21)
σ_θ	Std. Ambiguity Shock	$\mathcal{G}(0.1, 0.1)$	0.018, (00.015, 0.02)
ρ_{μ_Ψ}	AR(1) Invest. Tech Shock	$\mathcal{B}(0.5, 0.1)$	0.32, (0.28, 0.37)
ρ_θ	AR(1) Ambiguity Shock	$\mathcal{B}(0.5, 0.1)$	0.86, (0.84, 0.88)
Measurement errors			
$100\sigma_{\Delta,dy}$	output wedge	$\mathcal{G}(0.1, 1)$	0.36, (0.33, 0.39)
$100\sigma_{\Delta,\pi}$	inflation wedge	$\mathcal{G}(0.1, 1)$	0.16, (0.14, 0.17)
$100\sigma_{\Delta,u}$	unemployment wedge	$\mathcal{G}(0.1, 1)$	0.41, (0.37, 0.46)
Calibrated parameters			
β	Discount factor	0.9968	-
δ_k	Physical capital depreciation rate	2.5	-
χ	Calvo price stickiness	0.66	-
λ	Price Markup	1.2	-
ρ	Job survival probability	0.9	-
τ	Max. bargaining rounds per quarter	60	-
σ	Matching function elasticity	0.55	-
δ	Probability of bargaining breakup	0.19%	-
$400\bar{\mu}$	Output growth per capita	1.7	-
D	Replacement ratio	0.37	-
$100\eta_h$	Hiring costs	0.4	-
$100\eta_s$	Vacancy costs	0.03	-
$400\bar{\mu}_\Psi$	Investment per capital gowth rate	1.2	-
$400\bar{\pi}$	Inflation rate	2.5	-
g	Government consumption to output	0.2	-
b	Consumption habit	0.8	-
σ_a	Capacity utilization	0.11	-
a_I''	Investment adjustment cost	15.7	-
α	Capital share	0.26	-
θ	Technology diffusion	0.05	-

Table 1: Estimated and calibrated parameters. The priors $\mathcal{G}(\mu, \sigma), \mathcal{B}(\mu, \sigma)$ denote Gamma and Beta distributions with mean μ and standard deviation σ.

Variable		w^{μ_A}	w^{μ_Ψ}	w^R	w^f	**meas. error**
$y_t - y_{t-1}$	Output growth	36.4	30.9	0.7	32.0	-
$c_t - \log \Phi_t$	Consumption	52.4	5.0	0.2	42.4	-
$i_t - \log \Psi_t$	Investment	72.6	21.7	0.3	5.4	-
$v_t - \log \Phi_t$	Continuation values	53.8	6.0	0.1	40.0	-
π_t	Inflation rate	41.8	56.6	0.1	1.5	-
η_t	Hiring rate	38.7	31.6	0.9	28.9	-
$1 - l_t$	Unemployment rate	37.6	45.1	0.3	17.0	-
R_t	Nominal interest rate	37.8	54.7	6.3	1.2	-
$\overline{\Delta}_t^{(4)}(dy)$	Output growth wedge	-	-	-	18.5	81.5
$\overline{\Delta}_t^{(4)}(u)$	Unemployment wedge	-	-	-	16.3	83.7
$\overline{\Delta}_t^{(4)}(\pi)$	Inflation rate wedge	-	-	-	29.7	70.3

Table 2: Variance decomposition at the posterior modes. All values are in percent.

6 Understanding the role of ambiguity shocks

Table 2 provides the variance decomposition for key macroeconomic variables and the belief wedges. Despite substantial noise in the survey answers, the estimated model picks up a meaningful amount of common variation from the survey answers (around 20% on average), and has a substantial impact on key macroeconomic variables. In this section, we analyze in detail the mechanism through which ambiguity shocks propagate into the economy.

6.1 Belief wedges and the worst-case model

Figure 4 depicts the impulse responses for the ambiguity shock w_t^f. A one-standard deviation increase in ambiguity leads to a fall of about 2% in output growth on impact, and to almost a 1 percentage point increase in unemployment that peaks after about four quarters. Inflation increases in the moment of the impact of the shock as well. These responses are larger than those estimated using the reduced-form model from Section 3 and depicted in Figure 3 but they tell the same qualitative story. The bottom row of Figure 4 shows households become pessimistic about GDP growth, expecting even lower growth than the one predicted using the impulse response from the top left panel. Households also overpredict inflation and unemployment. All shifts in beliefs are again consistent with the reduced-form findings from Section 3.

The structural model allows to explain the economic mechanism underlying the role of the ambiguity shock. This shock affects households' concerns about model misspecification and therefore alters their worst-case model $\widetilde{P}$. In order to understand the impact of the ambiguity shock, it is therefore useful to distinguish between the impulse responses under the data-generating process P and under the worst-case model $\widetilde{P}$. The former impulse responses are those observed by the rational econometrician, while the latter are perceived by the household in the model.

Figure 5 compares both responses to the ambiguity shock w_t^f. After an increase in ambiguity,

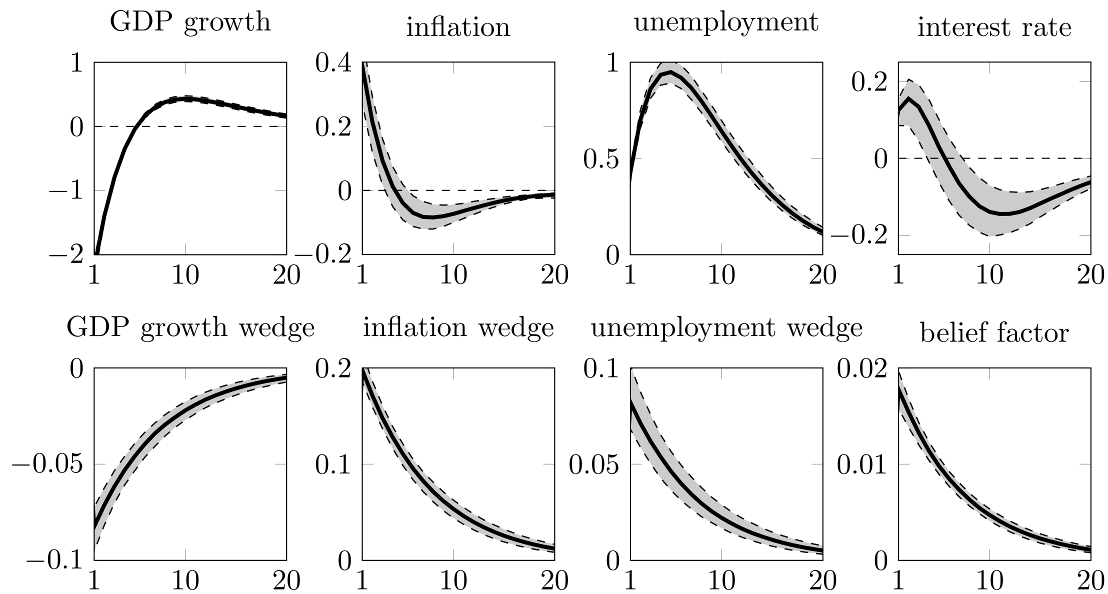

Figure 4: Bayesian impulse response functions to the belief shock w^f. Output growth, inflation rate and interest rate in annualized percent, and unemployment rate in percentage points. The solid lines indicated median estimates, while the dashed line correspond to the 10th and 90th percentile error bands. Horizontal axis in quarters.

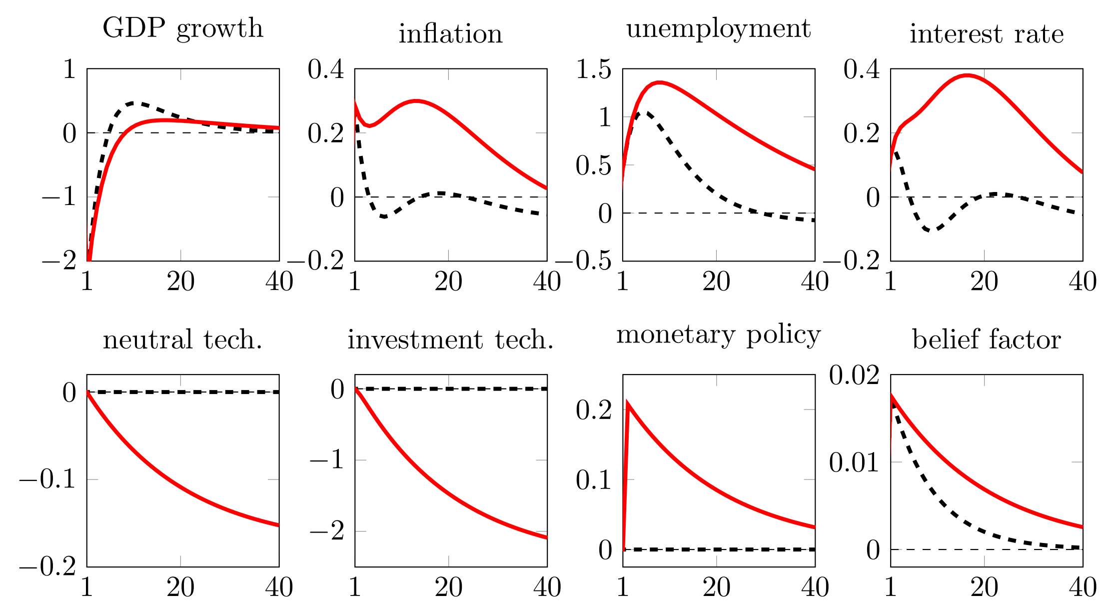

Figure 5: Impulse response functions to the belief shock w^f under the data-generating measure P (black dashed line) and the worst-case model $\widetilde{P}$ (red solid line). Impulse response functions evaluated at the mode of the posterior distribution. Horizontal axis in quarters.

the households' worst-case model becomes more pessimistically biased. In line with expression (10), the worst-case impulse responses are more persistent — the households expects the adverse effects

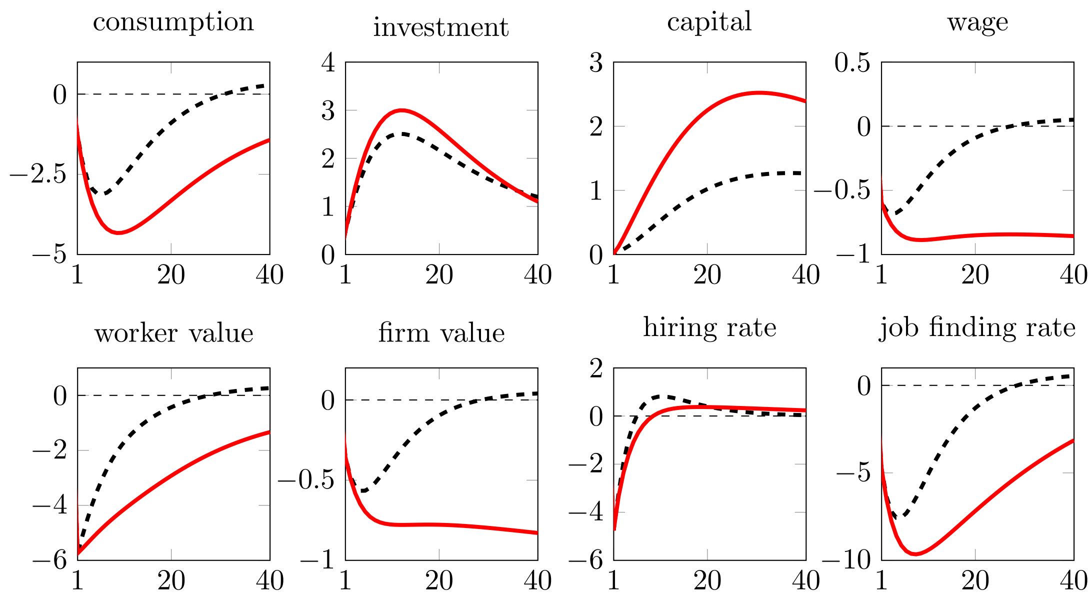

Figure 6: Impulse response functions to the belief shock w^f under the data-generating measure P (black dashed line) and the worst-case model $\widetilde{P}$ (red solid line). Impulse response functions evaluated at the mode of the posterior distribution. Horizontal axis in quarters.

of an ambiguity increase on the economy to persist longer.

The bottom row of Figure 5 depicts the impulse responses for the individual exogenous shocks in the model. The dashed line in the bottom right figure depicts the response of the belief process $\theta_t = f_t$ to the innovation w_t^f. The household indeed expects under the worst case model (red solid line) the ambiguity increase to be more persistent. On the other hand, because the individual exogenous shocks are uncorrelated, there is no response of the technology processes or the monetary policy shock to the innovation w_t^f — the dashed lines in the corresponding panels are flat.

The story under the worst-case model is very different and critical to the understanding of the endogenous response of the macroeconomy to the ambiguity shock. Under the worst-case model, the household believes that the shocks are correlated in an adverse way. An increase in ambiguity worsens the expectation of the household about the future path of the neutral and investment-specific technology, and the household also expects a monetary tightening.

This particular correlation structure arises because these three innovations to the exogenous processes all leads to a decrease in the continuation value V_t. In other words, times with low neutral and investment-specific productivity growth, and times in which the economy is hit by an exogenous monetary tightening through the shock w_t^R are bad times, with a low continuation value V_t. Moreover, the continuation value recursion (5) indicates that these bad times must be generated by low levels of current and future consumption under the households' worst-case model. The first panel Figure 6 indeed confirms this intuition — the household that faces an increase in ambiguity forecasts a large and very persistent drop in consumption. The second panel shows that the increase in ambiguity is also accompanied by a large increase in investment activity, generated

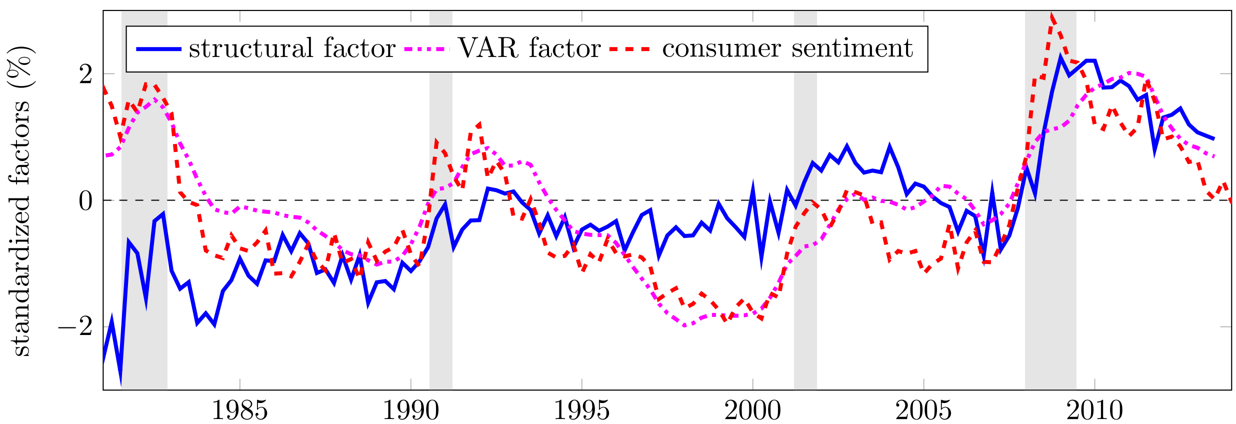

Figure 7: A comparison of the extracted ambiguity factor with the Michigan Survey measure of contemporaneous Consumer Sentiment. The solid blue line is the smoothed factor from the structural model, the purple dash-dotted line the smoothed factor from the reduced-form VAR model. All data series are standardized.

by the precautionary effect of the increase in pessimism about future growth.

6.2 Labor market dynamics

These pessimistic expectations interact in crucial ways through the frictional labor market. With search and matching rigidities, hiring and bargaining decisions are based on the value of the discounted future surplus generated by a match. Both firms and workers inherit the representative household's beliefs to make future forecasts when they compute their respective continuation values. Lower expected productivity and higher expected interest rates lower the value of the match from the perspective of the worst-case beliefs shared by the worker-firm pair. This lowers equilibrium hiring rates, and lower employment also implies lower output. Equilibrium wages also fall, reflecting the decline in the surplus that is particularly large and persistent under the worst-case model. All these effects are capture by the remaining responses in Figure 6.

This channel induced by fluctuations in household's ambiguity concerns is a novel and potent source of fluctuations in the labor market. The variance decomposition in Table 2 reveals that the ambiguity shocks drive a substantial portion of the overall variation in the labor market variables, for instance almost one third of the variation in the hiring rate.

The effect on the inflation rate comes from a balance of two forces. Lower contemporaneous aggregate demand pushes the intermediate goods producers that change prices to set them to lower levels. At the same time, expectations of lower productivity imply higher marginal costs and this pushes current and future prices upwards. At our current estimates, the net effect of an increase in ambiguity is a higher equilibrium inflation rate in the year after the impact of the ambiguity shock. At the same time, the response of the inflation wedge is positive, indicating that the worst-case model is biased toward an even higher inflation rate in the future.

Finally, in Figure 7 we plot the extracted series for the ambiguity factor obtained from the reduced-form and structural models, along with the Consumer Sentiment index reported by the Michigan Survey. All three series are highly correlated and attest to a consistent narrative of how ambiguity affects business cycle dynamics.

7 Conclusion

We develop a framework in which time-variation in ambiguity perceived by households generates fluctuations in aggregate dynamics of the macroeconomy. The framework is based on an extension of the robust preference model that introduces shocks to agents' concerns about model misspecification. We identify these *ambiguity shocks* using survey data from the University of Michigan Surveys of Consumers and the Survey of Professional Forecasters. We show that in the data and in an estimated business cycle model, the ambiguity shocks are a potent source of variation in labor market variables.

The structural interpretation of ambiguity shocks identified in our framework opens new directions for policy analysis under ambiguity. In parallel work, we study the implications of this framework for optimal monetary policy. A monetary authority facing households endowed with robust preferences infers that policy changes lead to endogenous changes in the worst-case model. The choice of optimal policy therefore involves explicit management of households' expectations by the monetary authority.

Appendix

A Distorted beliefs in the one-factor model

Let $(\Omega, \{\mathcal{F}_t\}_{t=0}^{\infty}, P)$ be the probability space generated by the innovations of model (1). The subjective probability measure $\widetilde{P}$ is formally defined by specifying a strictly positive martingale M with one-period increment

$$m_{t+1} = \frac{M_{t+1}}{M_t} = \exp\left(-\frac{1}{2}|k_t|^2 + k_t' w_{t+1}\right).$$

We then have $\widetilde{E}_t[w_{t+1}] = k_t$. Using the notation $x_t = (y_t', f_t)'$, the factor structure (2)–(3) of households' expectations is obtained by imposing the restriction

$$k_t = \overline{H} + HFx_t$$

where $F = (F_y, F_f)$ in an $1 \times n$ vector and H, $\overline{H}$ are $k \times 1$ vectors.

Let $\zeta_t = Zx_t$ be the vector of variables for which we have observable data on households' expectations where Z is a selection matrix. Here, we derive results for more general dynamics that is in line with the non-stationary model from Section 4. Specifically, we assume that

$$\begin{aligned}
\zeta_t &= Zx_t = Z\widehat{x}_t + z_t \\
\widehat{x}_{t+1} &= \psi_q + \psi_x \widehat{x}_t + \psi_w w_{t+1} \\
z_{t+1} - z_t &= \phi_q + \phi_x \widehat{x}_t + \phi_q w_{t+1}.
\end{aligned}$$

The process z_t introduces an additional component of the dynamics that has stationary growth rates. The factor model framework from Section 3 is obtained by setting the matrices $\overline{H}$, ψ_q, ψ_x, ψ_w and ϕ_q to zero, in which case

$$x_{t+1} = \psi_x x_t + \psi_w w_{t+1}$$

is a concise form for (1).

We are interested in τ-period belief wedges

$$\Delta_t^{(\tau)} = \widetilde{E}_t[\zeta_{t+1}] - E_t[\zeta_{t+1}].$$

Assume that

$$\begin{aligned}
E_t[\zeta_{t+\tau} - \zeta_t] &= G_\tau^x \widehat{x}_t + G_\tau^0 \\
\widetilde{E}_t[\zeta_{t+\tau} - \zeta_t] &= \widetilde{G}_\tau^x \widehat{x}_t + \widetilde{G}_\tau^0
\end{aligned}$$

where G_τ^x, G_τ^0, $\widetilde{G}_\tau^x$ and $\widetilde{G}_\tau^0$ are conformable matrix coefficients with initial conditions

$$G_\tau^0 = \widetilde{G}_\tau^0 = 0_{n\times 1} \qquad G_\tau^x = \widetilde{G}_\tau^x = 0_{n\times n}.$$

We can then establish a recursive formula for the expectations under the data-generating measure

$$
\begin{aligned}
G_{\tau}^{x}\widehat{x}_t + G_{\tau}^{0} &= E_t\left[\zeta_{t+\tau} - \zeta_t\right] = \\
&= E_t\left[Z\left(x_{t+1} - x_t\right) + G_{\tau-1}^{x}\widehat{x}_{t+1} + G_{\tau-1}^{0}\right] \\
&= G_{\tau-1}^{0} + \phi_q + \left(Z + G_{\tau-1}^{x}\right)\psi_q + \left[\left(Z + G_{\tau-1}^{x}\right)\psi_x + \left(\phi_x - Z\right)\right]\widehat{x}_t \\
&\quad + \left(\left(Z + G_{\tau-1}^{x}\right)\psi_w + \phi_w\right)E_t\left[w_{t+1}\right].
\end{aligned}
\tag{16}
$$

Since $E_t\left[w_{t+1}\right] = 0$, we obtain

$$
\begin{aligned}
G_{\tau}^{x} &= \left(Z + G_{\tau-1}^{x}\right)\psi_x + \left(\phi_x - Z\right) \\
G_{\tau}^{0} &= G_{\tau-1}^{0} + \phi_q + \left(Z + G_{\tau-1}^{x}\right)\psi_q.
\end{aligned}
$$

Under the subjective measure, the derivation is unchanged, except the last line in (16) that now involves the subjective expectation $\widetilde{E}_t\left[w_{t+1}\right] = \overline{H} + HF\widehat{x}_t$. Then

$$
\begin{aligned}
\widetilde{G}_{\tau}^{x} &= \left(Z + \widetilde{G}_{\tau-1}^{x}\right)\psi_x + \left(\phi_x - Z\right) + \left(\left(Z + \widetilde{G}_{\tau-1}^{x}\right)\psi_w + \phi_w\right)HF \\
\widetilde{G}_{\tau}^{0} &= \widetilde{G}_{\tau-1}^{0} + \phi_q + \left(Z + \widetilde{G}_{\tau-1}^{x}\right)\psi_q + \left(\left(Z + \widetilde{G}_{\tau-1}^{x}\right)\psi_w + \phi_w\right)\overline{H}
\end{aligned}
$$

Consequently

$$
\Delta_t^{(\tau)} = \left(\widetilde{G}_{\tau}^{x} - G_{\tau}^{x}\right)\widehat{x}_t + \widetilde{G}_{\tau}^{0} - G_{\tau}^{0}.
$$

In the case considered in Section 3 when $\overline{H}$, ψ_q, ψ_x, ψ_w and ϕ_q are all zero, we get explicit expressions

$$
\begin{aligned}
G_{\tau}^{x} &= Z\left(\psi_x\right)^{\tau} \\
G_{\tau}^{0} &= Z\sum_{i=0}^{\tau-1}\left(\psi_x\right)^{i}\psi_q = Z\left(I - \psi_x\right)^{-1}\left(I - \left(\psi_x\right)^{\tau}\right)\psi_q \\
\widetilde{G}_{\tau}^{x} &= Z\left(\psi_x + \psi_w HF\right)^{\tau} \\
\widetilde{G}_{\tau}^{0} &= Z\sum_{i=0}^{\tau-1}\left(\psi_x + \psi_w HF\right)^{i}\psi_q = Z\left(I - \left(\psi_x + \psi_w HF\right)\right)^{-1}\left(I - \left(\psi_x + \psi_w HF\right)^{\tau}\right)\psi_q.
\end{aligned}
$$

B Series expansion of the worst-case model

The linear approximation in this paper is an extension of the series expansion method used in Holmes (1995) or Lombardo (2010). Borovička and Hansen (2013, 2014) adapt the series expansion method to an approximation of models with robust preferences. Here, we further extend this methodology to derive a linear solution that allows for endogenously determined time-varying belief distortions. The critical step in the expansion lies in the joint perturbation of the shock vector w_t and the penalty process θ_t.

B.1 Law of motion

We start with the approximation of the model for the law of motion (4) with a sufficiently smooth policy rule ψ. We consider a class of models indexed by the scalar perturbation parameter q that scales the volatility of the shock vector w_t

$$
x_t\left(\mathsf{q}\right) = \psi\left(x_{t-1}\left(\mathsf{q}\right), \mathsf{q}w_t, \mathsf{q}\right) \tag{17}
$$

and assume that there exists a series expansion of x_t around $\mathsf{q} = 0$:

$$x_t(\mathsf{q}) \approx \bar{x} + \mathsf{q}x_{1t} + \frac{\mathsf{q}^2}{2}x_{2t} + \ldots.$$

The processes x_{jt}, $j = 0, 1, \ldots$ can be viewed as derivatives of x_t with respect to the perturbation parameter, and their laws of motion can be inferred by differentiating (17) j times and evaluating the derivatives at $\mathsf{q} = 0$, assuming that ψ is sufficiently smooth. Here, we focus only on the approximation up to the first order:

$$\begin{aligned} \bar{x} &= \psi(\bar{x}, 0, 0) \\ x_{1t} &= \psi_x x_{1t-1} + \psi_w w_t + \psi_q. \end{aligned} \tag{18}$$

Here, we assume that the equilibrium dynamics of x_t is stationary. Extensions to non-stationary environments are considered in Appendix B.6.

B.2 Continuation values

We now focus on the expansion of the continuation value recursion. Substituting the worst-case belief distortion (7) into the recursion (5) yields

$$V_t = u(x_t) - \frac{\beta}{\theta_t}\log E_t\left[\exp\left(-\theta_t V_{t+1}\right)\right]. \tag{19}$$

We are looking for an expansion of the continuation value

$$V_t(\mathsf{q}) \approx \bar{V} + \mathsf{q}V_{1t}. \tag{20}$$

In order to derive the solution of the continuation value, we are interested in expanding the following perturbation of the recursion:

$$V_t(\mathsf{q}) = u(x_t(\mathsf{q}), \mathsf{q}) - \beta\frac{\mathsf{q}}{\overline{\theta}(\bar{x} + x_{1t})}\log E_t\left[\exp\left(-\frac{\overline{\theta}(\bar{x} + x_{1t})}{\mathsf{q}}V_{t+1}(\mathsf{q})\right)\right]. \tag{21}$$

Here, we utilized the fact that $\theta_t = \overline{\theta}x_t \approx \overline{\theta}(\bar{x} + x_{1t})$. More importantly, the perturbation scales jointly the volatility of the stochastic processes for V_t and $u(x_t)$ with the magnitude of the penalty parameter θ_t. In particular, the penalty parameter in the perturbation of equation (5) becomes q/θ_t and decreases jointly with the volatility of the shock process. This assumption will imply that the benchmark and worst-case models do not converge as $\mathsf{q} \to 0$, and the linear approximation around a deterministic steady state yields a nontrivial contribution of the worst-case dynamics.

Using the expansion of the period utility function

$$u(x_t(\mathsf{q}), \mathsf{q}) \approx \bar{u} + \mathsf{q}u_{1t} = \bar{u} + \mathsf{q}(u_x x_{1t} + u_q)$$

we get the deterministic steady state (zero-th order) term by setting $\mathsf{q} = 0$:

$$\bar{V} = (1 - \beta)^{-1}\bar{u}.$$

The first-order term in the expansion is derived by differentiating (21) with respect to q and is given by the recursion

$$V_{1t} = u_{1t} - \beta \frac{1}{\overline{\theta}\left(\bar{x} + x_{1t}\right)} \log E_t \left[\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_{1t+1}\right)\right] \tag{22}$$

Since $\bar{x}$ is constant and x_{1t} has linear dynamics (18), we hope to find linear dynamics for V_{1t} as well. Since $u_t = u\left(x_t\right)$, we can make the guess that $V_t^i\left(\mathsf{q}\right) = V^i\left(x_t\left(\mathsf{q}\right), \mathsf{q}\right)$ which leads to the following expressions for the derivative of V_t:

$$V_{1t} = V_x x_{1t} + V_q.$$

Using this guess and comparing coefficients, equation (22) leads to a pair of algebraic equations for the unknown coefficients V_x and V_q:

$$\begin{aligned} V_x &= u_x - \frac{\beta}{2} V_x \psi_w \psi_w' V_x' \overline{\theta} + \beta V_x \psi_x \\ V_q &= u_q - \frac{\beta}{2} \overline{\theta} \bar{x} V_x \psi_w \psi_w' V_x' + \beta V_x \psi_q + \beta V_q \end{aligned}$$

The first from this pair of equations is a Riccati equation for V_x, which can be solved for given coefficients ψ_x and ψ_w.

B.3 Distortions

With the approximation of the continuation value at hand, we can derive the expansion of the one-period belief distortion m_{t+1} that defines the worst-case model relative to the benchmark model. As in (21), we scale the penalty parameter θ_t jointly with the volatility of the underlying shocks:

$$m_{t+1}\left(\mathsf{q}\right) = \frac{\exp\left(-\frac{1}{\mathsf{q}}\theta_t V_{t+1}\left(\mathsf{q}\right)\right)}{E_t\left[\exp\left(-\frac{1}{\mathsf{q}}\theta_t V_{t+1}\left(\mathsf{q}\right)\right)\right]} \approx m_{0,t+1} + \mathsf{q} m_{1,t+1}.$$

It turns out that in order to derive the correct first-order expansion, we are required to consider a second-order expansion of the continuation value

$$V_t\left(\mathsf{q}\right) \approx \bar{V} + \mathsf{q} V_{1t} + \frac{\mathsf{q}}{2} V_{2t},$$

although the term V_{2t} will be inconsequentil for subsequent analysis. Substituting in expression (20) and noting that $\bar{V}$ is a deterministic term, we can approximate m_{t+1} with

$$m_{t+1}\left(\mathsf{q}\right) \approx \frac{\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right)\left(V_{1t+1} + \frac{\mathsf{q}}{2} V_{2t+1}\right)\right)}{E_t\left[\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right)\left(V_{1t+1} + \frac{\mathsf{q}}{2} V_{2t+1}\right)\right)\right]}$$

Differentiating with respect to q and evaluating at $\mathsf{q} = 0$, we obtain the expansion

$$\begin{aligned} m_{0t+1} &= \frac{\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_{1t+1}\right)}{E_t\left[\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_{1t+1}\right)\right]} \\ m_{1t+1} &= -\frac{1}{2\overline{\theta}\left(\bar{x} + x_{1t}\right)} M_{0t+1}\left[V_{2t+1} - E_t\left[M_{0t+1} V_{2t+1}\right]\right] \end{aligned} \tag{23}$$

This expansion is distinctly different from the standard polynomial expansion familiar from the perturbation literature. First, observe that m_{0t+1} is not constant, as one would expect for a zeroth-order term, but nonlinear in V_{1t+1}. However, since $E_t\left[m_{0t+1}\right] = 1$ we can thus treat M_{0t+1} as a change of measure that will adjust the distribution of shocks that are correlated with m_{0t+1}. We will show that with Gaussian shocks, we can still preserve tractability. Further notice that $E_t\left[m_{1t+1}\right]=0$.

The linear structure of V_{1t} also has an important implication for the worst-case distortion constructed from m_{0t+1}. Substituting into (23) yields

$$m_{0t+1}=\frac{\exp\left(-\overline{\theta}\left(\bar{x}+x_{1t}\right)V_x\psi_w w_{t+1}\right)}{E_t\left[\exp\left(-\overline{\theta}\left(\bar{x}+x_{1t}\right)V_x\psi_w w_{t+1}\right)\right]}.$$

This implies that for a function $f\left(w_{t+1}\right)$ with a shock vector $w_{t+1}\sim N\left(0,I\right)$,

$$E_t\left[m_{0t+1}f\left(w_{t+1}\right)\right]\approx E_t\left[m_{t+1}f\left(w_{t+1}\right)\right]=\widetilde{E}_t\left[f\left(w_{t+1}\right)\right]$$

where, under the $\widetilde{P}$ (worst-case) measure, the vector w_{t+1} has the following distribution:

$$w_{t+1}\sim N\left(-\overline{\theta}\left(\bar{x}+x_{1t}\right)\left(V_x\psi_w\right)',I_k\right). \tag{24}$$

the mean of the shock is therefore time-varying and depends on the linear process x_{1t}.

B.4 Equilibrium conditions

We assume that equilibrium conditions in our framework can be written as

$$0=E_t\left[\widetilde{g}\left(x_{t+1},x_t,x_{t-1},w_{t+1},w_t\right)\right] \tag{25}$$

where $\widetilde{g}$ is an $n\times 1$ vector function and the dynamics for x_t is implied by (4). This vector of equations includes expectational equations like Euler equations of the robust household, which can be represented using worst-case belief distortions m_{t+1}. We therefore assume that we can write the j-th component of $\widetilde{g}$ as

$$\widetilde{g}^j\left(x_{t+1},x_t,x_{t-1},w_{t+1},w_t\right)=m_{t+1}^{\sigma_j}g^j\left(x_{t+1},x_t,x_{t-1},w_{t+1},w_t\right).$$

where $\sigma_j\in\{0,1\}$ captures whether the expectation in the j-th equation is under the household's worst-case model.[3] In particular, all nonexpectational equations and all equations not involving agents' preferences have $\sigma_j=0$. System (25) can then be written as

$$0=E_t\left[\mathbb{M}_{t+1}g\left(x_{t+1},x_t,x_{t-1},w_{t+1},w_t\right)\right]$$

where $\mathbb{M}_{t+1}=\operatorname{diag}\left\{m_{t+1}^{\sigma_1},\ldots,m_{t+1}^{\sigma_n}\right\}$ is a diagonal matrix of the belief distortions, and g is independent of the robustness parameter θ_t. As in Borovička and Hansen (2013), the zero-th and first-order expansions are

$$\begin{aligned}0&=E_t\left[\mathbb{M}_{0t+1}g_{0t+1}\right]=g_{0t+1}\\0&=E_t\left[\mathbb{M}_{0t+1}g_{1t+1}\right]+E_t\left[\mathbb{M}_{1t+1}g_{0t+1}\right]=E_t\left[\mathbb{M}_{0t+1}g_{1t+1}\right]\end{aligned}$$

where the last equality follows from $E_t\left[m_{1t+1}\right]=0$.

[3] The generalization to multiple agents with potentially heterogeneous concerns for robustness is straightforward, see the construction in Borovička and Hansen (2013).

For the first-order derivative of the equilibrium conditions, we have

$$0 = E_t \left[\mathbb{M}_{0t+1} g_{1t+1} \right] \tag{26}$$

The first-order term in the expansion of g_{t+1} is given by

$$\begin{aligned} g_{1t+1} &= g_{x+}x_{1t+1} + g_x x_{1t} + g_{x-}x_{1t-1} + g_{w+}w_{t+1} + g_w w_t + g_q = \\ &= \left[\left(g_{x+}\psi_x + g_x \right) \psi_x + g_{x-} \right] x_{1t-1} + \left[\left(g_{x+}\psi_x + g_x \right) \psi_w + g_w \right] w_t + \\ &\quad + \left(g_{x+}\psi_x + g_{x+} + g_x \right) \psi_q + g_q + \left(g_{x+}\psi_w + g_{w+} \right) w_{t+1} \end{aligned} \tag{27}$$

where symbols x_+, x, x_-, w_+, w, q represent partial derivatives with respect to $x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t$ and q, respectively. Given the worst-case distribution of the shock vector (24), we can write

$$\widetilde{E}_t \left[w_{t+1} \right] = - \left(V_x \psi_w \right)' \overline{\theta} \left[\left(\bar{x} + \psi_q \right) + \psi_x x_{1t-1} + \psi_w w_t \right]$$

Let $[A]^i$ denote the i-th row of matrix A. Notice that

$$\left[g_{x+}\psi_w + g_{w+} \right]^i \left(V_x \psi_w \right)' \overline{\theta}$$

is a $1 \times n$ vector. Construct the $n \times n$ matrix $\mathbb{E}$ by stacking these row vectors for all equations $i = 1, \ldots, n$:

$$\mathbb{E} = \text{stack} \left\{ \sigma_i \left[g_{x+}\psi_w + g_{w+} \right]^i \left(V_x \psi_w \right)' \overline{\theta} \right\}$$

which contains non-zero rows for expectational equations under the worst-case model. Using matrix $\mathbb{E}$, we construct the conditional expectation of the last term in g_{1t+1} in (27). In particular

$$\begin{aligned} 0 &= E_t \left[\mathbb{M}_{0t+1} g_{1t+1} \right] = \\ &= \left[\left(g_{x+}\psi_x + g_x \right) \psi_x + g_{x-} \right] x_{1t-1} + \left[\left(g_{x+}\psi_x + g_x \right) \psi_w + g_w \right] w_t + \\ &\quad + \left(g_{x+}\psi_x + g_{x+} + g_x \right) \psi_q + g_q - \mathbb{E} \left[\left(\bar{x} + \psi_q \right) + \psi_x x_{1t-1} + \psi_w w_t \right] \end{aligned}$$

Equation (26) is thus a system of linear second-order stochastic difference equations. There are well-known results that discuss the conditions under which there exists a unique stable equilibrium path to this system (Blanchard and Kahn (1980), Sims (2002)). We assume that such conditions are satisfied. Comparing coefficients on x_{1t-1}, w_t and the constant term implies that

$$0 = \left(g_{x+}\psi_x + g_x - \mathbb{E} \right) \psi_x + g_{x-} \tag{28}$$

$$0 = \left(g_{x+}\psi_x + g_x - \mathbb{E} \right) \psi_w + g_w \tag{29}$$

$$0 = \left(g_{x+}\psi_x + g_{x+} + g_x \right) \psi_q + g_q - \mathbb{E} \left(\bar{x} + \psi_q \right) \tag{30}$$

These equations need to be solved for ψ_x, ψ_w, ψ_q and V_x where

$$V_x = u_x - \frac{\beta}{2} V_x \psi_w \psi_w' V_x' \overline{\theta} + \beta V_x \psi_x$$

and

$$\mathbb{E} = \text{stack} \left\{ \sigma_i \left[g_{x+}\psi_w + g_{w+} \right]^i \left(V_x \psi_w \right)' \overline{\theta} \right\}. \tag{31}$$

B.5 Special case: θ_t is an exogenous AR(1) process

In the application, we consider a special case that restricts θ_t to be an exogenous AR(1) process. With a slight abuse in notation, this restriction can be implemented by replacing the vector of variables x_t with $(x_t', f_t)'$ where f_t is a scalar AR(1) process representing the time-variation in the concerns for robustness as an exogenously specified 'belief' shock:

$$f_{t+1} = (1-\rho_f)\,\bar{f} + \rho_f f_t + \sigma_f w_{t+1}^f. \tag{32}$$

The dynamics of the model then satisfies

$$x_t = \psi\left(x_{t-1}, w_t, f_t\right) \tag{33}$$

with steady state $\left(\bar{x}', \bar{f}\right)'$. The vector $\overline{\theta}$ in (6) is then partitioned as $\overline{\theta}' = \left(\overline{\theta}_x', \overline{\theta}_f\right) = (0_{1\times n-1}, 1)$ and thus $\theta_t = f_t$. Constructing the first-order series expansion of (33), we obtain

$$\begin{pmatrix} x_{1t+1} \\ f_{1t+1} \end{pmatrix} = \begin{pmatrix} \psi_q \\ 0 \end{pmatrix} + \begin{pmatrix} \psi_x & \rho_f \psi_{xf} \\ 0 & \rho_f \end{pmatrix}\begin{pmatrix} x_{1t} \\ f_{1t} \end{pmatrix} + \begin{pmatrix} \psi_w & \sigma_f \psi_{xf} \\ 0 & \sigma_f \end{pmatrix}\begin{pmatrix} w_{t+1} \\ w_{t+1}^f \end{pmatrix}$$

where w_{t+1} and w_{t+1}^f are uncorrelated innovations. The matrices ψ_x and ψ_w thus do not involve any direct impact of the dynamics of the belief shock f_{1t} and the matrix ψ_{xf} captures how the dynamics of f_{1t} influences the dynamics of endogenous state variables.

Let us further assume that the system (25) represents the equilibrium restrictions of the model *except* equation (32). In this case, the function g does not directly depend on f. Repeating the expansion of the equilibrium conditions from Section B.4 and comparing coefficients on x_{t-1}, f_{t-1}, w_t and the constant term yields the set of conditions for matrices ψ_x, ψ_w, ψ_{xf} and ψ_q:

$$\begin{aligned} 0 &= \left(g_{x+}\psi_x + g_x\right)\psi_x + g_{x-} && (34)\\ 0 &= \left(g_{x+}\rho_f\psi_{xf} - \mathbb{E}\right) + \left(g_{x+}\psi_x + g_x\right)\psi_{xf} && (35)\\ 0 &= \left(g_{x+}\psi_x + g_x\right)\psi_w + g_w && (36)\\ 0 &= \left(g_{x+}\psi_x + g_{x+} + g_x\right)\psi_q + g_q - \mathbb{E}\bar{f} && (37) \end{aligned}$$

with

$$\begin{aligned} V_x &= u_x + \beta V_x \psi_x && (38)\\ V_f &= u_f - \frac{\beta\overline{\theta}}{2}\left(V_f^2\sigma_f^2 + 2V_x\psi_{xf}\sigma_f^2 V_f + V_x\left(\sigma_f^2\psi_{xf}\psi_{xf}' + \psi_w\psi_w'\right)V_x'\right) && (39)\\ &\quad + \beta\left(V_f\rho_f + V_x\psi_{xf}\rho_f\right) \\ \mathbb{E} &= \text{stack}\left\{\sigma^i\left[g_{x+}\psi_{xf}\sigma_f^2\left(V_f + V_x\psi_{xf}\right) + \left(g_{x+}\psi_w + g_{w+}\right)\psi_w' V_x'\right]^i\right\}\overline{\theta}. && (40) \end{aligned}$$

This set of equations is the counterpart of equations (28)–(31) and can be solved sequentially. First, notice that equations (34) and (36) can be solved for ψ_x and ψ_w, and these coefficients are not impacted by the dynamics of f_t. But the equilibrium dynamics of x_t is affected by movements in f_t through the coefficient ψ_{xf}. The coefficient $\rho_f\psi_{xf}$ introduces an additional component in the time-varying drift of x_t, while $\sigma_f\psi_{xf}$ is an additional source of volatility arising from the shocks to household's concerns for robustness.

We solve this set of equations by backward induction. First, we use (28), (31) and (38) to find the no-ambiguity solution for ψ_x, ψ_w, V_x. Then we postulate that (33) is in fact a time-dependent law of motion

$$x_t = \psi^t\left(x_{t-1}, w_t, f_t\right)$$

with terminal condition at a distant date T

$$x_T = \psi^T\left(x_{T-1}, w_T, 0\right).$$

This corresponds to assuming that starting from date T, ambiguity is absent in the model. Plugging this guess to the set of equilibrium conditions, we obtain the set of algebraic equations

$$\begin{aligned}
0 &= \left(g_{x+}\psi_{xf}^{t+1}\rho_f - \mathbb{E}^{t+1}\right) + \left(g_{x+}\psi_x + g_x\right)\psi_{xf}^t && (41)\\
V_f^t &= u_f - \frac{\beta\overline{\theta}}{2}\left(\left(V_f^{t+1}\sigma_f\right)^2 + 2V_x\psi_{xf}^{t+1}\sigma_f^2 V_f^{t+1} + V_x\left(\sigma_f^2\psi_{xf}^{t+1}\left(\psi_{xf}^{t+1}\right)' + \psi_w\psi_w'\right)V_x'\right) && (42)\\
&\quad +\beta\rho_f\left(V_f^{t+1} + V_x\psi_{xf}^{t+1}\right) &&\\
\mathbb{E}^{t+1} &= \left[g_{x+}\psi_{xf}^{t+1}\left(V_f^{t+1} + V_x\psi_{xf}^{t+1}\right)\sigma_f^2 + \left(g_{x+}\psi_w + g_{w+}\right)\psi_w' V_x'\right]\overline{\theta}. && (43)
\end{aligned}$$

Equation (41) can then be solved for

$$\psi_{xf}^t = \left(g_{x+}\psi_x + g_x\right)^{-1}\left(\mathbb{E}^{t+1} - g_{x+}\psi_{xf}^{t+1}\rho_f\right) \tag{44}$$

Iterating backwards on equations (42)–(44) backward until convergence yields the stationary solution of the economy with ambiguity as a long-horizon limit of an economy where ambiguity vanishes at a distant T. The system converges as long as its dynamics are stationary under the worst-case model. Once we find the limit $\lim_{t\to-\infty}\mathbb{E}^t = \mathbb{E}$, we can also determine

$$\psi_q = \left(g_{x+}\psi_x + g_{x+} + g_x\right)^{-1}\left(\mathbb{E}\bar{f} - g_q\right).$$

B.6 Nonstationary models

Consider the nonstationary dynamics introduced in Section 4.3. When the period utility function is given by (13), i.e., $u\left(x_t\right) = \widehat{u}\left(\widehat{x}_t\right) + z_t$, then using the guess for the continuation value (14), we can rewrite equation (19) as

$$\widehat{V}\left(\widehat{x}_t\right) = \widehat{u}\left(\widehat{x}_t\right) - \frac{\beta}{\theta_t}\log E_t\left[\exp\left(-\theta_t\left(\widehat{V}\left(\widehat{x}_{t+1}\right) + (1-\beta)^{-1}\phi\left(\widehat{x}_t, w_{t+1}\right)\right)\right)\right]$$

with $\widehat{u}\left(\widehat{x}_t\right) = \log\widehat{C}\left(x_t\right)$. The first-order expansion of ϕ yields

$$\begin{aligned}
\bar{z}_{t+1} - \bar{z}_t &= \phi\left(\bar{x}, 0\right)\\
z_{1t+1} - z_{1t} &= \phi_q + \phi_x\widehat{x}_{1t} + \phi_w w_{t+1}
\end{aligned}$$

where $\bar{x}$ is the steady state of $\widehat{x}_t$. We can now proceed as in the stationary case except using the expansion of functions $\widehat{u}$ and $\widehat{V}$. We have

$$\bar{V} = (1-\beta)^{-1}\left[\bar{u} + \beta(1-\beta)^{-1}\phi\left(\bar{x}, 0\right)\right]$$

and

$$\widehat{V}_{1t} = V_x \widehat{x}_{1t} + V_q$$

with

$$\begin{aligned} V_x &= u_x + \beta \left[V_x \psi_x + (1-\beta)^{-1} \phi_x \right] - \frac{\beta}{2} \left| V_x \psi_w + (1-\beta)^{-1} \phi_w \right|^2 \overline{\theta} \\ V_q &= u_q + \beta \left[V_q + V_x \psi_q + (1-\beta)^{-1} \phi_q \right] - \frac{\beta}{2} \overline{\theta} \bar{x} \left| V_x \psi_w + (1-\beta)^{-1} \phi_w \right|^2 . \end{aligned}$$

The zero-th order distortion is consequently given by

$$m_{0t+1} = \frac{\exp \left(-\overline{\theta} \left(\bar{x} + x_{1t} \right) \left(V_x \psi_w + (1-\beta)^{-1} \phi_w \right) w_{t+1} \right)}{E_t \left[\exp \left(-\overline{\theta} \left(\bar{x} + x_{1t} \right) \left(V_x \psi_w + (1-\beta)^{-1} \phi_w \right) w_{t+1} \right) \right]}$$

so that under the worst-case model,

$$w_{t+1} \sim N \left(-\overline{\theta} \left(\bar{x} + x_{1t} \right) \left(V_x \psi_w + (1-\beta)^{-1} \phi_w \right)', I_k \right).$$

We are still solving the set of equations (28)–(30) but now with V_x and $\mathbb{E}$ given by

$$\begin{aligned} V_x &= u_x + \beta \left[V_x \psi_x + (1-\beta)^{-1} \phi_x \right] - \frac{\beta}{2} \left| V_x \psi_w + (1-\beta)^{-1} \phi_w \right|^2 \overline{\theta} \\ \mathbb{E} &= \text{stack} \left\{ \sigma_i \left[g_{x+} \psi_w + g_{w+} \right]^i \left(V_x \psi_w + (1-\beta)^{-1} \phi_w \right)' \overline{\theta} \right\}. \end{aligned}$$

In the special case described in Section B.5, the belief shock f_t is modeled as an exogenous AR(1) process. The first-order dynamics of the stochastic growth rate can be expressed as

$$z_{1t+1} - z_{1t} = \phi_q + \phi_x \widehat{x}_{1t} + \phi_{xf} f_{1t} + \phi_w w_{t+1} + \phi_{wf} w^f_{t+1}.$$

The only modications appearing in the model solution are those related to the continuation value recursion and the shock distortion in $\mathbb{E}$. Specifically,

$$\begin{aligned} V_x &= u_x + \beta \left[V_x \psi_x + (1-\beta)^{-1} \phi_x \right] \\ V_f &= u_f + \beta \left(\rho_f V_f + \rho_f V_x \psi_{xf} + (1-\beta)^{-1} \phi_{xf} \right) \\ &\quad - \frac{\beta \overline{\theta}}{2} \left| V_x \psi_w + (1-\beta)^{-1} \phi_w \right|^2 - \frac{\beta \overline{\theta}}{2} \left| V_x \psi_{xf} \sigma_f + V_f \sigma_f + (1-\beta)^{-1} \phi_{wf} \right|^2 \\ \mathbb{E} &= \text{stack} \left\{ \sigma^i \left[\left(g_{x+} \psi_w + g_{w+} \right) \left(V_x \psi_w + (1-\beta)^{-1} \phi_w \right)' \right]^i \right\} \overline{\theta} \\ &\quad + \text{stack} \left\{ \sigma^i \left[g_{x+} \psi_{xf} \sigma_f \left(V_f \sigma_f + V_x \psi_{xf} \sigma_f + (1-\beta)^{-1} \phi_{wf} \right) \right]^i \right\} \overline{\theta} \end{aligned}$$

In the recursive form, V_f and $\mathbb{E}$ can be solved by iterating on the pair of equations

$$
\begin{aligned}
V_f^t &= u_f + \beta\left(\rho_f V_f^{t+1} + \rho_f V_x \psi_{xf}^{t+1} + (1-\beta)^{-1}\phi_{xf}\right) \\
&\quad -\frac{\beta\overline{\theta}}{2}\left|V_x\psi_w + (1-\beta)^{-1}\phi_w\right|^2 - \frac{\beta\overline{\theta}}{2}\left|V_x\psi_{xf}^{t+1}\sigma_f + V_f^{t+1}\sigma_f + (1-\beta)^{-1}\phi_{wf}\right|^2 \\
\mathbb{E}^{t+1} &= \text{stack}\left\{\sigma^i\left[(g_{x+}\psi_w + g_{w+})\left(V_x\psi_w + (1-\beta)^{-1}\phi_w\right)'\right]^i\right\}\overline{\theta} \\
&\quad +\text{stack}\left\{\sigma^i\left[g_{x+}\psi_{xf}^{t+1}\sigma_f\left(V_f^{t+1}\sigma_f + V_x\psi_{xf}^{t+1}\sigma_f + (1-\beta)^{-1}\phi_{wf}\right)\right]^i\right\}\overline{\theta}.
\end{aligned}
$$

together with equation (44) which remained unchanged.

C Data

To be written.

D Estimation of the one-factor model

Recall that we estimate the model

$$
\begin{aligned}
\begin{pmatrix} y_{t+1} \\ f_{t+1} \end{pmatrix} &= \begin{pmatrix} \psi_y & \psi_{yf}\rho_f \\ 0 & \rho_f \end{pmatrix}\begin{pmatrix} y_t \\ f_t \end{pmatrix} + \begin{pmatrix} \psi_{yw} & \psi_{yf}\sigma_f \\ 0 & \sigma_f \end{pmatrix}\begin{pmatrix} w_{t+1}^y \\ w_{t+1}^f \end{pmatrix} \\
\Delta_{t+1}^{(4)} &= \psi_{\Delta f} f_{t+1} + \sigma_\Delta \varepsilon_{t+1}^\Delta
\end{aligned}
$$

We estimate the model using a Metropolis–Hastings algorithm. We take five chains with different initial draws and make 20,000 draws in each chain. The first 10,000 draws of each chain are dropped.

The priors and posterior parameter estimates are reported in Table 3. The inverse-gamma priors on the standard deviations shrink the estimates away from zero in order to prevent overfitting. The means of the priors for the measurement error standard deviations are scaled by the standard deviations of the wedges.

The variance decomposition at the estimated mode of the parameters is reported in Table 4.

E Details on the New-Keynesian model

E.1 Alternating offer bargaining

As in Hall and Milgrom (2008) and Christiano et al. (2015), we assume that wages are determined by the alternating offer bargaining protocol of Rubinstein (1982) and Binmore et al. (1986).

At the start of period t, l_t matches are determined. Each worker then engages in bilateral bargaining with a wholesaler firm over the current wage rate ξ_t. The bargaining is conditional on all other period t matches as well as beliefs about future wage bargains.

We suppose bargaining takes place across τ subperiods within the period, where τ is even. Conditional on all previous offers having been rejected, the firm makes a wage offer every odd subperiod, while the worker makes a wage offer every even subperiod. The recipient can accept or reject an offer. If the recipient rejects an offer, she can end negotiations or plan to make a counteroffer in the next subperiod. In the latter case, the bargaining breaks down with probability δ. Following the estimates of Christiano et al. (2015), we set $\delta = 0.19$. We assume that when indifferent between accepting and rejecting an offer, an agent accepts it.

Parameter		Prior D,Mean,Std	Posterior Mean, 90% HPD
VAR coefficients			
$\psi_{y,ii}$		$\mathcal{B}(0.7, 0.2)$	
$\psi_{y,ij}, i \neq j$		$N(0, 2)$	
$100\psi_{w,ii}$		$\mathcal{IG}(1, 0.5)$	
$100\psi_{w,ij}, i \neq j$		$N\,(0, 2)$	
$\psi_{yf,dy}$		$N\,(0, 1)$	-0.29 (-0.47,-0.14)
$\psi_{yf,u}$		$N\,(0, 1)$	0.07 (0.03,0.13)
$\psi_{yf,\pi}$		$N\,(0, 1)$	0.08 (0.03,0.13)
$\psi_{yf,R}$		$N\,(0, 1)$	-0.03 (-0.06, 0.01)
Factor coefficients			
ρ_f		$\mathcal{B}\,(0, 5, 0.1)$	0.90 (0.86,0.93)
$100\sigma_f$		$\mathcal{IG}\,(0.5, 0.2)$	0.56 (0.31,0.80)
H_π		$N\,(0, 1)$	1.27 (0.62,1.89)
H_u		$N\,(0, 1)$	2.26 (1.55,3.03)
H_R		$N\,(0, 1)$	-0.45 (-1.95,1.07)
H_f		$N\,(0, 1)$	-1.25 (-1.77,-0.69)
Measurement errors			
$100\sigma_{\Delta,dy}$	GDP growth wedge	$\mathcal{IG}(0.7, 0.3)$	0.20 (0.18,0.22)
$100\sigma_{\Delta,\pi}$	inflation wedge	$\mathcal{IG}(3.3, 1)$	1.16 (1.04,1.26)
$100\sigma_{\Delta,u}$	unemployment wedge	$\mathcal{IG}(0.4, 2)$	0.19 (0.17,0.20)

Table 3: Prior and posterior estimates for reduced form model. The priors $\mathcal{IG}\,(\mu, \sigma)$ and $\mathcal{B}\,(\mu, \sigma)$ denote inverse Gamma and Beta distributions with mean μ and standard deviation σ.

Variable		w^{dy}	w^{u}	w^{π}	w^{R}	w^{f}	meas. error
dy_t	GDP growth	53.0	30.1	6.6	7.4	7.4	-
u_t	Unemployment rate	6.4	30.3	13.7	24.2	25.4	-
π_t	Inflation rate	3.0	14.4	68.5	10.9	3.2	-
R_t	Nominal interest rate	3.0	13.6	37.7	42.5	3.1	-
$\overline{\Delta_t}^{(4)}(dy)$	GDP growth wedge	-	-		-	55.5	44.5
$\overline{\Delta_t}^{(4)}(u)$	Unemployment wedge	-	-		-	43.9	56.1
$\overline{\Delta_t}^{(4)}(\pi)$	Inflation rate wedge	-	-		-	7.5	92.5

Table 4: Variance decomposition at the posterior modes for reduced form model. All values are in percent.

In odd subperiod $\iota < \tau$ of period t, the firm chooses the minimum wage offer $\xi_{\iota,t}$ that will not be rejected by the worker. In particular, $\xi_{\iota,t}$ satisfies

$$W_{\iota,t} = \max\left\{U_{\iota,t}, \delta U_{\iota,t} + (1-\delta)\left(D_t/\tau + W_{\iota+1,t}\right)\right\} \tag{45}$$

where

$$W_{\iota,t} = \xi_{\iota,t} + \widetilde{E}_t\left[\frac{S_{t+1}}{S_t}\left(\left(\rho + (1-\rho)\, j_{t+1}\right) W_{t+1} + (1-\rho)\left(1-j_{t+1}\right) U_{t+1}\right)\right]$$

denotes the value to a worker of accepting the offer $\xi_{\iota,t}$. The worker's outside option $U_{\iota,t}$ satisfies

$$U_{\iota,t} = \frac{\tau - \iota + 1}{\tau} D_t + \widetilde{E}_t\left[\frac{S_{t+1}}{S_t}\left(j_{t+1} W_{t+1} + \left(1-j_{t+1}\right) U_{t+1}\right)\right].$$

Similarly, in even subperiod $\iota < \tau$ of period t, the worker chooses the maximum wage offer $\xi_{\iota,t}$ that will not be rejected by the firm. In particular, $\xi_{\iota,t}$ satisfies

$$J_{\iota,t} = \max\left\{0, (1-\delta)\left(-\gamma_t + J_{\iota+1,t}\right)\right\} \tag{46}$$

where

$$J_{\iota,t} = \frac{\tau - \iota + 1}{\tau}\vartheta_t - \xi_{\iota,t} + \rho \widetilde{E}_t\left[\frac{S_{t+1}}{S_t} J_{t+1}\right]$$

denotes the value to a firm of accepting the wage offer $\xi_{\iota,t}$.

In the final subperiod τ, the worker makes a final offer. The worker offers the highest possible wage that the firm does not reject, which implies the condition

$$J_{\tau,t} = 0. \tag{47}$$

Starting from the terminal condition (47), we can solve the model backwards using the indifference conditions (45) and (46). This yields the condition

$$J_t = \beta_1\left(W_t - U_t\right) - \beta_2\gamma_t + \beta_3\left(\vartheta_t - D_t\right)$$

where $\beta_i = \alpha_{i+1}/\alpha_i$, with α_i defined as follows:

$$\begin{aligned}
\alpha_1 &= 1 - \delta + (1-\delta)^\tau \\
\alpha_2 &= 1 - (1-\delta)^\tau \\
\alpha_3 &= \alpha_2\frac{1-\delta}{\delta} - \alpha_1 \\
\alpha_4 &= \frac{1-\delta}{2-\delta}\frac{\alpha_2}{\tau} + 1 - \alpha_2.
\end{aligned}$$

E.2 Growth rate and functional forms

The model has two sources of growth - neutral and investment-specific technological progress. For a balanced growth path in the nonstochastic steady state, we require that the elements $\{\phi_t, s_t, \kappa_t, \gamma_t, G_t, D_t\}$ grow at the same rate

$$\Phi_t = \Psi_t^{\frac{\alpha}{1-\alpha}} A_t$$

in steady state. We thus set

$$\left(\phi_t, s_t, \kappa_t, \gamma_t, G_t, D_t\right)' = \left(\phi, s, \kappa, \gamma, G, D\right)' \Omega_t$$

where Ω_t is defined as

$$\Omega_t = \Phi_{t-1}^{0.05} \Omega_{t-1}^{0.95}$$

which is consistent with the estimates found in Christiano et al. (2015)

We assume the cost for capacity utilization are given by

$$a_u\left(u_t^K\right) = \frac{0.11}{2}\varphi\left(u_t^K\right)^2 + 0.89\varphi u_t^K + \varphi\left(\frac{0.11}{2} - 1\right)$$

where φ is chosen so that the steady state value of u_t^K is one. In addition, we assume that investment adjustment costs are given by

$$a_I\left(\frac{I_t}{I_{t-1}}\right) = \frac{1}{2}\left[\exp\left(\sqrt{15.7}\left(\frac{I_t}{I_{t-1}} - \mu \times \mu_\Psi\right)\right) + \exp\left(-\sqrt{15.7}\left(\frac{I_t}{I_{t-1}} - \mu \times \mu_\Psi\right)\right)\right] - 1$$

where μ and μ_Ψ denote the unconditional growth rates of Φ_t and Ψ_t respectively. Both cost functions are taken from Christiano et al. (2015).

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