ECON-GA 4091/GB 5091 — Computational Dynamics
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Spring 2023

Class Project

The class project consists of two deliverables.

1. A writeup in the form of a short research paper. The envisioned length is perhaps 10 pages but having concise, well-argued, and rigorous content is much more important than the number of pages. Any codes used in implementation should be delivered separately.

2. A 20–30 minute presentation of the results with a Q&A given in Week 7 of the course.

Below is a list of suggested topics. If you feel you have another project that you would like to engage in, feel free to suggest it but the project must be focused on computational dynamics preferably using the tools we learned in class. Suggested projects not on the list are subject to approval.

The projects are deliberately not formulated as homework problems with a clear 'correct' answer. They are meant for you to engage in a rigorous formulation of the research question, setting up the quantitative framework, executing a combination of theoretical and empirical work, summarizing the findings, and providing conclusions. The research paper should follow this template.

You do not need to strictly adhere to the problem setup outlined in the project descriptions, they are provided as a starting point. If you find a better or more convenient way to model the problem, I encourage you to do so. The models that you develop in the project will not provide a perfect fit to data—on the contrary. Understanding and explaining the limitations of the models is as important as providing a good fit.

1 Testing the capital asset pricing model

We want to study the prediction power of the so-called capital asset pricing model (CAPM) using the set of Euler equations for returns

$$1 = E_t\left[\frac{S_{t+1}}{S_t}R^n_{t+1}\right], \qquad n = 1, \ldots, N.$$

We already study the model under the assumption that the SDF is generated by a CRRA utility marginal rate of substitution

$$\frac{S_{t+1}}{S_t} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

One of the reasons why the model was not particularly successful in explaining average excess returns across assets is the low correlation between consumption growth and asset returns.

The CAPM assumes that the SDF is of the form

$$\frac{S_{t+1}}{S_t} = \beta \left(R^W_{t+1} \right)^{-\gamma}$$

where R^W_{t+1} is the return on investors' wealth. There is a tight connection between the returns on investors' wealth and consumption growth because of the impact of wealth effects on consumption. But perhaps consumption is poorly measured or subject to sluggish adjustments not accounted for in the model, so the return on wealth is a more immediate measure to be used in the SDF.

The goal is to test this theory using data from Kenneth French's data library

https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Proxy the return on investors' wealth with the return on the aggregate stock market R^m_{t+1}. Estimate the parameters β and γ, and provide the test of overidentifying restrictions. For the estimated parameter values, plot a scatter plot with average excess returns for the portfolios used in estimation on the horizontal axis, and the predicted returns implied by the covariance between the SDF and the excess returns on the vertical axis:

$$E\left[R^m_{t+1} - R^f_{t+1}\right] = -\left(E\left[\frac{S_{t+1}}{S_t}\right]\right)^{-1} Cov\left[\frac{S_{t+1}}{S_t}, R^m_{t+1} - R^f_{t+1}\right].$$

Explore a variety of choices of returns and instruments in order to investigate how well the model performs. Start with the risk-free rate and the excess return on the stock market as a baseline case. Then add a business-cycle variable, for example the unemployment rate or its cyclical component (unemployment rate minus the noncyclical rate of unemployment (FRED code NROU)) as an instrument, or other instruments of your choosing.

Then add more returns from the Kenneth French library. You can add the Fama/French 3 factors, 6 (or 25) Portfolios Formed on Size and Book-to-Market, or a variety of industry portfolios. All constitute valid returns or excess returns. Experiment with monthly and annual returns, if available. The goal is to understand how well the model performs vis-à-vis these returns, and how the estimated parameter change across specifications.

2 Valuation with disaster risk

An appealing idea how to rectify the failure of the Mehra and Prescott (1985) model to fit the empirical risk-free rate and risk premium is to add the possibility of disasters (see, for example, Rietz (1988), Barro (2006), Gourio (2012), or Gabaix (2012)). Imagine we extend the Mehra and Prescott (1985) model with an extra 'disaster' state in which consumption growth $C_{t+1}/C_t \ll 1$.

Clearly, this disasters state must be sufficiently unlikely because we do not observe such large disasters in U.S. data. However, we can perhaps argue that the U.S. economy has simply been 'lucky' because similar disasters do occasionally happen in other countries, and investors are aware that such a disaster may happen despite the fact that it has not happened since at least the Great Recession.

The idea for adding the disaster state is that it may potentially substantially increase the riskiness of the economy, so that the risk premium, as a measure of required compensation for holding risky assets, increases to empirically plausible values.

Experiment with the calibration of the transition matrix $\mathbf{P}$ and consumption growth rate matrix Γ_C in such an extended model, combined with the range of 'plausible' preference parameters that Mehra and Prescott (1985) used.

The goal is to find a model calibration that fits the unconditional risk-free rate and unconditional risk premium in the data, and perhaps other moments of interest as well. Provide a discussion on how the choice of particular parameters influences the results. For the preferred parameter values, provide a discussion of the model results, including (but not limited to) the unconditional risk premium and risk-free rate, as well as their conditional values in the alternative states, and the stationary distribution of the Markov state.

3 Search model over the business cycle

We want to study to which extent can the McCall (1970) search model be extended to account for business cycle fluctuations in unemployment. For this purpose, consider the following extension of the McCall (1970) model. There is an exogenous finite-state Markov chain x_t with a transition matrix $\mathbf{P}$. You can think about a two-state Markov chain encoding recession and expansion periods but you can also resort to a more refined specification.

The model can be, for example, modified as follows. First, every period, the worker receives an offer with probability $\lambda(x_t)$ (high in expansions, low in recessions). Not receiving an offer has the same consequences as rejecting an offer.

The idea is to find a calibration of the model that meaningfully fits the business cycle properties of U.S. labor market. Consider, for example, a calibration at the quarterly frequency. The calibration of $\mathbf{P}$ could be conducted using data on U.S. expansions and recessions, available from the NBER website

https://www.nber.org/research/business-cycle-dating.

In particular, we can compute how likely it is for an economy to stay in a recession next period if it is in a recession in this period. $\lambda(x_t)$ can be calibrated to business cycle differences in the job opening rate (FRED code JTSJOR). The model will feature a state-dependent reservation wage $\bar{w}(x_t)$, probability of accepting an offer $1 - F(\bar{w}(x_t))$, as well as probability of finding the job $f(x_t) = \lambda(x_t)[1 - F(\bar{w}(x_t))]$.

We now want to simulate the pool of unemployed workers. Denote u_t the unemployment rate, representing the share of labor force searching for jobs. To make things simple, assume that every period, u_t increases by n newly unemployed workers, while at the same time, $u_t f(x_t)$ unemployed workers find new jobs. This gives the law of motion

$$u_{t+1} = u_t + n - u_t f(x_t).$$

Now simulate the path of the unemployment rate, either via a random draw of the path of the states x_t, or by feeding it for x_t the actual time series of recessions and expansions for the U.S. economy.

The goal is to find a calibration of the model that would quantitatively resemble the dynamics of U.S. unemployment rate.

4 Learning about recessions

The NBER business cycle dating committee uses a range of data, econometric techniques and judgement to determine periods during which the economy is in a recession. Imagine the committee is using the following state space model

$$\begin{aligned} x_{t+1} &= \rho x_t + \sigma w_{t+1} \\ y_t &= G x_t + v_t \end{aligned}$$

where x_t is a scalar continuous variable reflecting the unobservable state of the economy, and y_t is an $m \times 1$ vector of observable variables (economic quantities like GDP growth, unemployment, inflation, etc.). The committee uses data y_t to make the best prediction $\hat{x}_t$ of the state of the economy, and announces that the economy is in a recession when $\hat{x}_t < R$ where R is a predetermined 'recession' threshold.

The actual recession periods are given by data, available from the NBER website

https://www.nber.org/research/business-cycle-dating.

Denote r_t to be the indicator function that is equal to one whenever the U.S. economy is in a recession. The model above makes a prediction $\hat{r}_t = \mathbf{1}\{\hat{x}_t < R\}$.

The research problem is to find the data for y_t and parameters of the state space model that provide the 'best' fit to the recession announcements, in the sense that the time series of predicted $\hat{r}_t$ is as close as possible to the r_t series.

References

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