ECON-GA 4091/GB 5091 — Computational Dynamics
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Problem Set 1

Due on Friday, February 3, 11 am, via Brightspace

For each of the problems, submit your answers and code that generates the results. A Jupyter notebook is fine, as long as the results and code are well organized, with appropriate discussion. Group discussion is encouraged but everybody has to produce code and writeup individually. You can use and adapt the available codes from Github if you wish.

1 Evaluation of asset pricing restrictions using GMM

We study the estimation of preference parameters from Euler equations in the form

$$E_t\left[\frac{S_{t+1}}{S_t}R^n_{t+1}\right] = 1, \qquad n = 1, \ldots, N,$$

where the stochastic discount factor is given by the marginal rate of substitution of an investor with CRRA preferences:

$$\frac{S_{t+1}}{S_t} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

Some of the Euler equations will come in the form of excess return restrictions

$$E_t\left[\frac{S_{t+1}}{S_t}\left(R^n_{t+1} - R^f_{t+1}\right)\right] = 0$$

where R^f_{t+1} is another return of interest.

Consumption C_t are real consumption expenditures per capita or another suitable quantity downloaded from FRED. Other macroeconomic quantities (inflation, instruments, ...) can also be downloaded from FRED. Returns are from the Kenneth French data library, appropriately deflated to real returns.

Before you start, I recommend to study the **Data processing** section in the Jupyter notebook prepared for this problem. The notebook is called jupyter/asset_pricing_gmm.ipynb, stored on the Github site:

https://github.com/jborovicka/nyu-computational-dynamics

The section describes in detail the data choices and necessary pre-preprocessing. The Python code pre-processes all data from scratch—a less time-intensive approach may be to prepare all data manually in a spreadsheet, and then use the final data for estimation directly.

For each of the following questions, estimate the required parameters, and find the p-value for the J-test of overidentifying restrictions. Do the data reject the model? After you provide results for all three questions, provide a brief comparison of the results. How do the estimates differ and why?

Question 1.1 Use data at quarterly frequency, appropriately accumulating monthly returns. Use the risk-free rate (RF) and the excess return on the stock market in excess of the risk-free rate (Mkt-RF) as asset returns for the moment conditions. Fix $\beta = 0.99$ and estimate γ.

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Question 1.2 Use data at annual frequency, extending the sample as far back as possible. Annual returns are readily available in the file Kenneth French data library, so no accumulation is required. Use the risk-free rate (RF), the excess return on the stock market in excess of the risk-free rate (Mkt-RF), and the excess returns on the SMB and HML portfolios as asset returns for the moment conditions. Estimate β and γ.

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Question 1.3 Use data at quarterly frequency, appropriately accumulating monthly returns. Use the risk-free rate (RF) and the excess return on the stock market in excess of the risk-free rate (Mkt-RF). Apart from using these unconditional moments, also instrument both equations using the unemployment rate as instrument z_t (so that you have four moment conditions in total). Estimate β and γ.

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2 Yield curve in the Mehra and Prescott economy

Mehra and Prescott (1985) proposed an economy with underlying uncertainty described by a two-state Markov chain x_t with transition matrix

$$\mathbf{P} = \begin{bmatrix} \phi & 1-\phi \\ 1-\phi & \phi \end{bmatrix}.$$

Consumption growth is modeled as a stationary process

$$\frac{C_{t+1}}{C_t} = \exp\left(g_C\left(x_t, x_{t+1}\right)\right)$$

represented by the matrix Γ_C with elements

$$[\Gamma_C]_{ij} = \exp\left(g_C\left(x_t = e_i, x_{t+1} = e_j\right)\right).$$

Preferences are of the CRRA form with time preference β and relative risk aversion γ, implying the SDF of the form

$$\frac{S_{t+1}}{S_t} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} = \beta \exp\left(-\gamma g_C\left(x_t, x_{t+1}\right)\right)$$

represented by the matrix Γ_S with elements

$$[\Gamma_S]_{ij} = \beta \exp\left(-\gamma g_C\left(x_t = e_i, x_{t+1} = e_j\right)\right) = \beta\, [\Gamma_C]_{ij}^{-\gamma}$$

where the $-\gamma$ power applies elementwise.

Use the same parameterization as in Mehra and Prescott (1985), with

$$\Gamma_C = \begin{bmatrix} 1+\mu+\delta & 1+\mu-\delta \\ 1+\mu+\delta & 1+\mu-\delta \end{bmatrix}$$

so that state e_1 is the high-growth state, state e_2 is the low-growth state. Calibrate annual parameters $\mu = 0.018$, $\delta = 0.036$, $\phi = 0.43$.

The Matlab code available at matlab/asset_valuation_Markov_chain.m on Github:

https://github.com/jborovicka/nyu-computational-dynamics

models the valuation of a claim on the aggregate endowment. You can use this code as a starting point, or write your own code in Python. In this problem, you are asked to compute the term structure of interest rates that emerges from the model.

Question 2.1 A zero-coupon bond with maturity T is an asset that pays 1 unit T periods ahead. The time-t price of this zero-coupon bond therefore is

$$Q_t^{[T]} = E_t\left[\frac{S_{t+T}}{S_t} G_{t+T}\right] = E_t\left[\frac{S_{t+T}}{S_t} \cdot 1\right].$$

Use the recursive valuation formula

$$Q_t^{[T]} = E_t\left[\frac{S_{t+1}}{S_t} Q_{t+1}^{[T-1]}\right]$$

to compute the prices of these bonds for maturities $T = 1, \ldots, 20$. From these prices, compute the yields

$$y_t^{[T]} = -\frac{1}{T} \log Q_t^{[T]}.$$

Plot the term structure of interest rates, i.e., $y_t^{[T]}$ as a function of T. The term structure will depend on the current state at time t, i.e., you will get two functions, one for $x_t = e_1$ and another for $x_t = e_2$.

Repeat the exercise for the following parameter values:

- $\beta = 0.99$, $\gamma = 2$
- $\beta = 0.96$, $\gamma = 2$

- $\beta = 0.99$, $\gamma = 4$

Compare the results and attempt to provide economics reasoning for how the results differ across the parameterizations. ■

References

Mehra, Rajnish and Edward C. Prescott (1985) "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15 (2), 145–161.