ECON-GA 4091/GB 5091 — Computational Dynamics
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Problem Set 3

Due on Monday, February 20, 9:30 am, via Brightspace

For each of the problems, submit your answers and code that generates the results. A Jupyter notebook is fine, as long as the results and code are well organized, with appropriate discussion. Group discussion is encouraged but everybody has to produce code and writeup individually. You can use and adapt the available codes from Github if you wish.

1 Campbell–Shiller decomposition

The Campbell and Shiller (1988) decomposition is an approximation of an accounting identity (under a transversality condition) that relates the logarithm of the current price dividend ratio to present discounted value of future dividend growth rates and future returns:

$$z_t = \sum_{j=1}^{\infty} \rho^j g_{t+j} - \sum_{j=1}^{\infty} \rho^j r_{t+j} \tag{1.1}$$

where

$$z_t = \log \frac{Q_t}{G_t} \qquad g_{t+1} = \log \frac{G_{t+1}}{G_t} \qquad r_{t+1} = \log R_{t+1}$$

and the constant ρ is given by

$$\rho = \frac{\exp(\bar{z})}{\exp(\bar{z}) + 1}$$

where $\bar{z}$ is the logarithm of the steady state price dividend ratio, which we can set to $\bar{z} = \log E[Q_t/G_t]$.

We are interested in understanding whether movements in the price-dividend ratio are more associated with changes in future returns or future growth rates. For that, we compute the covariance of equation (1.1) with z_t, to obtain

$$Var(z_t) = Cov\left(z_t, \sum_{j=1}^{\infty} \rho^j g_{t+j}\right) + Cov\left(z_t, -\sum_{j=1}^{\infty} \rho^j r_{t+j}\right)$$

and, dividing by $Var(z_t)$,

$$1 = \frac{Cov\left(z_t, \sum_{j=1}^{\infty} \rho^j g_{t+j}\right)}{Var(z_t)} + \frac{Cov\left(z_t, -\sum_{j=1}^{\infty} \rho^j r_{t+j}\right)}{Var(z_t)}.$$

In order to implement this equation empirically, we need to truncate the infinite sum with a finite horizon J

$$1 = \frac{Cov\left(z_t, \sum_{j=1}^{J} \rho^j g_{t+j}\right)}{Var\left(z_t\right)} + \frac{Cov\left(z_t, -\sum_{j=1}^{J} \rho^j r_{t+j}\right)}{Var\left(z_t\right)}. \tag{1.2}$$

Question 1.1 Download the **U.S. Stock Markets 1871-Present and CAPE Ratio** data from Robert Shiller's website.

http://www.econ.yale.edu/~shiller/data.htm

These are monthly data for the U.S. stock market since 1871. The relevant columns are real price, real dividend, and real total return price (this is the cumulative return, the one-period return is the ratio of adjacent cells, as you can infer from the formulas entered in the spreadsheet). The dividends are annnualized, so you should convert them back to monthly values.

Using the available data, compute the two terms on the right-hand side of (1.2) for $J = 1, \ldots, 300$ months. Are price-dividend ratios associated more with fluctuations in dividend growth or fluctuations in future returns? ■

References

Campbell, John Y. and Robert J. Shiller (1988) "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1, 195–228.