ECON-GA 4091 — Computational Dynamics

NYU, Spring 2023

Jaroslav Borovička

Syllabus

Professor:	**Jaroslav Borovička**	jaroslav.borovicka@nyu.edu
	office hours	19W 4th Street, office 714 by appointment
Teaching Assistant:	**Brandon Kaplowitz**	bgk258@nyu.edu
	office hours	by appointment
Lecture:	KMEC, room 4–110	Mon, Wed 9:30 – 10:45 am
Recitation:	KMEC, room 4–110	Fri 11 – 12:15 pm

1 Course description

The solution of state-of-the-art quantitative models in economics and finance often requires numerical implementation on a computer. In this course, we will explore a range of computational methods used to solve such quantitative dynamic problems: Local approximations of equilibria in models for macro-policy analysis, global methods for approximating solutions to nonlinear decision problems, filtering techniques used in learning and estimation, discretization methods popular in financial economics, as well as neural network models used in data science.

We will use a hands-on approach. Each week, we will present a new method and apply it to the solution of a specific economic model. We will show in detail how to move from the mathematical specification of the model to the elaboration of a solution algorithm and finally to its Python implementation. This pedagogical approach will also give us the opportunity to illustrate basic techniques widely applicable in scientific computing, including functional approximation, optimization, numerical and symbolic differentiation, and quadrature.

2 Course requirements and grading

The weekly problem sets and a final project are designed to reinforce learning by requiring students to independently implement each newly acquired methodology.

2.1 Course objectives

1. Learn how to structure dynamic problems in ways suitable for implementation on a computer.
2. Become familiar with popular computational methods and their implementation in Python or another scientific computing language.
3. Understand advantages, disadvantages, and caveats of each method.

2.2 Assessment

Course performance will be assessed by means of weekly problem sets and a short final project.

1. The problem sets will account for **60%** of the final grade. While you are strongly encouraged to discuss homework in small groups, everybody has to submit their own answers and numerical implementation.
2. The final project will account for the remaining **40%**. You will work in groups of 2–3 people, submit a coauthored writeup and code, and present the outcome of the project in class during the exam week. The final project will be assigned at the beginning of week 4 of the course. The purpose of the provide is to tackle a numerical implementation of a particular economic model, and assess its implications.

The final grade will be a letter grade, computed from the cumulative points received from the problem sets and the final project. These points will be computed in accordance with the weights described and standardized to a 0–100 scale. Grades will then be assigned according to the following formula:

92–100	85–91	78–84	72–77	66–71	60–65	55–59	50–54	43–49	≤ 42
A	A–	B+	B	B–	C+	C	C–	D	F

3 Course outline

Among other resources, we make use of the QuantEcon lectures, available at

https://python.quantecon.org/.

- *Python Programming for Economics and Finance* is a useful starting point for those of you who plan to use Python for computational work in the course but had little experience with it so far:

  https://python-programming.quantecon.org/.

- *Quantitative Economics with Python* [**QEP**] provides basic economic modeling

  https://python.quantecon.org/

- *Advanced Quantitative Economics with Python* [**Advanced QEP**] discusses more advanced topics

  https://python-advanced.quantecon.org/

Week 1: Markov chains and asset pricing

Valuation of financial claims on a discrete state space. Modeling uncertainty. Probability matrices, expectations, and stochastic discounting. The Perron–Frobenius Theorem and asymptotic behavior.

- *Textbook*: Ljungqvist and Sargent (2020), Chapter 2 (Sections 2.2–2.3, Markov chains), Chapters 14 and 15 (asset pricing).
- *GMM*: Hansen (1982), Hansen (2008).
- *Asset pricing applications*: Lucas (1978), Hansen and Singleton (1982), Hansen and Singleton (1983), Mehra and Prescott (1985).
- QuantEcon: **QEP** Topic 3 (linear algebra), Topic 25 (finite Markov chains), Topics 74–76 (asset pricing applications in finite state models). **AdvancedQEP** Topics 34–35 (more advanced asset pricing applications).

Week 2: Value function iteration in search problems

Recursive formulation and the Bellman operator. Discretization of a continuous state space, vectorization. Quadrature methods and root solving. Infinite horizon problems as limit of finite horizon ones.

- *Textbook*: Ljungqvist and Sargent (2020), Chapter 7 (Section 7.1–7.4, search problems). Judd (1998), Chapter 12 (numerical dynamic programming).

- *Search problems*: McCall (1970).
- QuantEcon: **QEP** Topic 33–38 (search problems).

Week 3: Perturbation methods in valuation and macroeconomic dynamics

Linear state-space models. Impulse response functions. Steady state and linearization in the neighborhood of the steady state. Series expansion methods. Linear approximation of return decomposition. Real business cycle and new-Keynesian models.

- *Textbook*: Ljungqvist and Sargent (2020), Chapter 2 (Sections 2.4–2.6, linear dynamics), Chapters 5 and 6 (linear-quadratic dynamic programming). Stokey et al. (1989), Chapter 6 (linear approximation). Judd (1998), Chapter 13 (perturbation methods). Holmes (1995) (series expansion methods). Galí (2008), Chapters 2 and 3 (real business cycle and new Keynesian models).
- *Series expansion methods*: Lombardo (2010), Borovička and Hansen (2014).
- *Linearization of valuation equations*: Campbell and Shiller (1988).
- *Perturbation in general equilibrium models*: Blanchard and Kahn (1980), Sims (2002), Kydland and Prescott (1982), Smets and Wouters (2007).
- QuantEcon: **QEP** Topics 61–66 (linear quadratic control), Topics 69–70 (stability). **AdvancedQEP** Topics 16–23 (dynamic linear economies).

Week 4: Filtering methods in learning

Inference problem. State-space representation of models with unobserved states. The Kalman filter.

- *Textbook*: Ljungqvist and Sargent (2020), Chapter 2 (Sections 2.7–2.9, Kalman filter), Chapter 7 (Section 7.7–7.8 learning in search).
- *Linear filtering*: Friedman (1957), Kalman (1960), Muth (1960), Jovanovic (1979).
- QuantEcon: **QEP** Topic 31 (Kalman filter), Topic 56 (search with learning).

Week 5: Finite difference methods in derivative pricing

Brownian motion and Itô processes. Differential equation for the price of a derivative. Time and space discretization. Implicit and explicit schemes for solving discretized problems.

- *Textbook*: Duffie (2001), Chapters 5, 6, and 8 (continuous-time asset pricing).
- *Numerical methods*: Holmes (2007), Candler (2001), Judd (1998), Chapter 10 (finite-difference methods).
- *Derivative pricing*: Black and Scholes (1973), Merton (1973), Cox et al. (1979).

Week 6: Q-learning in decision problems

McCall (1970) model revisited. Consumption-saving problem. Bayesian learning and convergence. Reinforcement learning.

- *Textbook*: Sutton and Barto (2018), Goodfellow et al. (2016), Bishop (2006).
- QuantEcon: **QEP** Topic 39 (job search application).

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