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The Composition of Market Participants and Asset Dynamics

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DISCUSSION OVERVIEW

Summary of the paper

- theoretical and empirical aims

Empirical results

- relationship between data and model predictions

Model solution

- construction of decision rules does not correspond to a rational expectations equilibrium

MODEL

Endowment economy with dividend and labor income

- incomplete markets: two sources of risk, one traded risky asset

Investors endowed with Epstein–Zin preferences

- heterogeneous risk aversion $\implies$ wealth distribution becomes a state vector

Short-selling constraints

- endogenous stock market participation

TARGETS

Distinction between aggregate and stockholder consumption variation

Variation in stock market entry and exit

- in good times, more risk averse agents enter

Cyclicality of quantity and price of consumption risk

- largely constant price of risk
  - positive shock $\Longrightarrow$ entry of more risk averse agents $\Longrightarrow$ heightened average risk aversion ($\nearrow$ price of risk)
  - positive shock $\Longrightarrow$ consumption shares shift toward less risk averse agents ($\searrow$ price of risk)
  - first effect somewhat stronger
- countercyclical quantity of risk
  - procyclical aggregate consumption risk (larger share of riskier financial income in expansions)
  - strongly countercyclical stockholder consumption risk (dtto in recessions, due to exit of more risk averse investors)

EMPIRICAL EVIDENCE

Datasets

- Survey of Income and Participation Program (SIPP)
- Consumer Expenditure Survey (CEX)
- Survey of Consumer Finances (SCF) for risk aversion proxies

Results

- Association between business cycle and entries to/exits from the stock market by risk aversion
- Negative association between quantity of risk for stockholders and business cycle, positive association for aggregate consumption

DETERMINANTS OF ENTRIES AND EXITS FROM SIPP DATA (TABLE 5)

Dependent variable:	Entry$_{i,t}$		Exit$_{i,t}$	
	(1)	(2)	(3)	(4)
S_t/Y_t	0.003	-0.019	-0.007*	0.018**
	(1.21)	(-1.67)	(-2.10)	(2.38)
$S_t/Y_t \times \gamma_{i,t}$		0.039*		-0.042***
		(2.10)		(-3.13)
$\gamma_{i,t}$	-0.303***	-0.375***	0.257***	0.335***
	(-8.57)	(-5.62)	(10.97)	(7.86)

As we move from a recession to expansion (S_t/Y_t increases) and back, more risk averse agents are last to join but also first to leave.

- expect a positive coefficient on $S_t/Y_t \times \gamma_{i,t}$ both for entries and exits
- in the data, coefficients on entry and exit have opposite signs but almost the same magnitudes
- run the same regression on model generated data!

EMPIRICALLY ESTIMATED AMOUNT OF RISK (FIGURE 6)

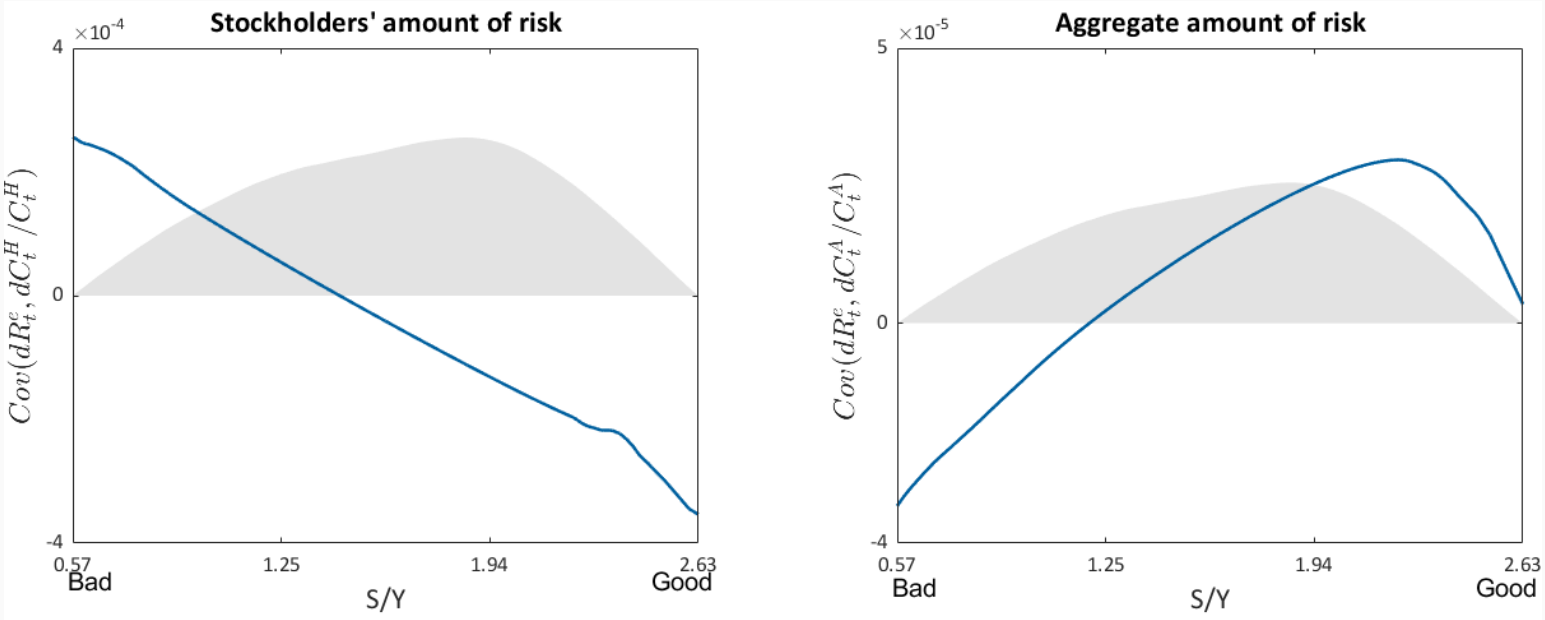

Figure 6: **Empirically estimated Amount of risk**

This figure plots the empirically estimated conditional covariance of equity returns with stockholders' (Left) and aggregate (Right) consumption growth using the stock market capitalization to aggregate non-financial income ratio (S/Y). The bold solid lines are the nonparametric estimate of conditional covariance based on the Epanechnikov kernel estimation at monthly frequency. The shaded backgrounds represent the rescaled kernel density of the conditioning variable. A detailed description of the data is in the online appendix A.3. The result using the consumption-wealth ($\widehat{cay}$) by Lettau and Ludvigson (2001) is in the online appendix figure OA.1.

- quantity of risk negative in a substantial part of the state space, which would predict negative risk premia

PRICE OF RISK DYNAMICS (TABLE 6)

Dependent variable	Independent variable			Adj. R^2
	S_t/Y_t	C_t^H/C_t^A	p_t	
Panel B: Price of risk dynamics				
C_t^H/C_t^A	0.030***			0.110
	(4.23)			
p_t	0.016***			0.073
	(3.60)			
$\sum_{i\in H} C_{i,t}/\sum_{i\in H}(C_{i,t}/\gamma_{i,t})$		-3.977***	5.098***	0.037
		(-4.33)	(3.50)	

- consumption-weighted risk aversion correctly predicted by stockholder's consumption share and participation
- repeat these regressions on model simulated data again!

Investor's decision problem leads to the HJB equation

$$
\begin{aligned}
0 \;=\; & \frac{\tilde{\delta}(1-\gamma)V_t}{1-\psi^{-1}}\left(\tilde{\delta}^{\psi-1}((1-\gamma)V_t)^{-\theta\psi+\psi-1}V_{x,t}^{1-\psi}\psi^{-1}-1\right)+((r_{f,t}+\nu)X_t+Y_t)V_{x,t} \\
& -\frac{\lambda_t^2 V_{x,t}^2}{2V_{xx,t}}+\mu_y Y_t V_{y,t}+\frac{1}{2}\sigma_y^2 Y_t^2 V_{yy,t}-\frac{\lambda_t V_{x,t}\rho_t\sigma_y Y_t V_{xy,t}}{V_{xx,t}}-\frac{\rho_t^2\sigma_y^2 Y_t^2 V_{xy,t}^2}{2V_{xx,t}}+\sum_{j=1}^{N-1}\mu_{w_{j,t}}w_{j,t}V_{w_j,t} \\
& +\frac{1}{2}\sum_{j=1}^{N-1}\sigma_{w_{j,t}}^2 w_{j,t}^2 V_{w_jw_j,t}-\frac{\sum_{j=1}^{N-1}\rho_{w_j,t}\sigma_{w_{j,t}}w_{j,t}\sigma_t V_{w_jx,t}(\lambda_t V_{x,t}+\rho_t\sigma_y Y_t V_{xy,t})}{\sigma_t V_{xx,t}} \\
& -\frac{(\sum_{j=1}^{N-1}\rho_{w_j,t}\sigma_{w_{j,t}}w_{j,t}\sigma_t V_{w_jx,t})^2}{2\sigma_t^2 V_{xx,t}}+\sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_y Y_t V_{w_jy,t} \\
& +\sum_{j\neq k}\rho_{w_j,w_k,t}\sigma_{w_{j,t}}\sigma_{w_{k,t}}w_{j,t}w_{k,t}V_{w_jw_k,t}+\frac{{l_t}^2}{2\sigma_t^2 V_{xx,t}} \qquad \text{(A.1)}
\end{aligned}
$$

$r_{f,t}$, λ_t, $\mu_{w_{j,t}}$ and $\sigma_{w_{j,t}}$ are all functions of the aggregate state $(D_t, Y_t, \mathbf{w}_t)$ that are determined in equilibrium

- high-dimensional nonlinear problem (with occasionally binding constraints on top of that)

MODEL SOLUTION

Authors 'guess and verify' the value function of the form

$$V_t = \frac{\left(a + \sum_{j=1}^{N-1} c_j w_{j,t}\right)(X_t + bY_t)^{1-\gamma}}{1-\gamma}$$

and obtain (under non-binding constraints and in a special limit)

$$\begin{aligned} a &= \left(\tilde{\delta}^{1-\psi}\psi\left(\left(-r_{f,t} - \nu - \frac{\lambda_t^2}{2\gamma}\right)\frac{1-\psi^{-1}}{\delta} + 1\right)\right)^{-\frac{1}{\theta\psi}} \\ c_j &= 0, \quad b = \frac{1}{r_{f,t} + \nu + \lambda_t \rho_{s,t}\sigma_y - \mu_y} \end{aligned}$$

- since $r_{f,t}$, λ_t are not constant, a and b cannot be constant either
- guess is not valid

INTERPRETATION OF THE SOLUTION

The value function and decision rules of individual agents are computed as if agents believed $r_{f,t}$ and λ_t are constant forever.

- this would be true in an iid growth economy but not in this model
- somewhat resembles anticipated utility (see Cogley and Sargent (2008) for a discussion)

These decision rules are subsequently aggregated to determine market-clearing prices

- but this does not constitute a rational expectations equilibrium

The authors provide the equity premium formula

$$E_t[dR_t^e] = \underbrace{\frac{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{s,t}^i}{\gamma_i}) ds}}_{\textit{Price of risk}} \times \underbrace{Cov_t(dR_t^e, \frac{d\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds})}_{\textit{Amount of risk}} \tag{27}$$

- price of risk a harmonic weighted mean of risk aversions
- quantity of risk covariance of consumption growth with returns
- standard CCAPM formula under CRRA utility

With Epstein–Zin preferences, continuation values should show up.

- under the 'iid' assumption built into individual decision rules, EZ reduces to CRRA

FINDING A PROPER SOLUTION METHOD

The model features 30 types of investors indexed by risk aversion.

- whole wealth distribution is a state vector

Challenges in finding a meaningful approximation

- treatment of binding/non-binding constraints
- simplifying forecasting rules
  - Krusell and Smith (1998) type algorithms
  - ignoring variation in future risk-free rates and Sharpe ratios (anticipated utility approach) is an extreme simplification
- moments being matched are pretty delicate, which requires careful analysis

E.g., specify a 2-agent economy where 1 type participates occasionally.

- solve accurately and using approximation to compare

SUMMARY

The project is interesting and has a lot of potential

- time-varying quantity of risk and roughly constant price of risk would help discriminate among 'workhorse' models (habits, long-run risk, disasters) $\implies$ this is critical

It is hard to say now how different the predictions in a full REE would be.

- qualitative intuition is sound and should still go through
- Epstein–Zin utility SDF can be very sensitive to belief specification
- Epstein–Zin utility with predictable state variables breaks the tight link between consumption and returns
  - unobservable continuation values make the empirical validation more challenging

Tighter model comparison to data

- run same regressions
- provide impulse response functions to model shocks, perhaps accompanied by empirical counterparts