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# INFORMED TRADING AND INTERTEMPORAL SUBSTITUTION: THE LIMITS OF THE NO-TRADE THEOREM

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No-trade theorem(s) (*Milgrom and Stokey (1982)* and subsequent extensions) show that

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then subsequent release of (private or public) information cannot lead to retrading.

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## First-order conditions

$$\sum_i \lambda^i \left(u'\right)' \left(c_t^i; \theta_t\right) = \mu_t\left(Y_t\left(\theta^t\right)\right) \qquad \mu \text{ is the L.M. on the constraint}$$

- completely static, separable problem.

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which could have been chosen by the planner

- but wasn't $\implies$ contradiction

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## Non-separable preferences

- this paper
- habit formation, Epstein–Zin, …

An agent has risk-aversion-dominating preferences when, $\forall t$ and $\forall C$

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- $c_t$ impacts marginal utility of consumption in other states and periods
- it may make sense to correlate $c_t$ with consumption in other periods
- additional information (e.g., about future states) can lead to retrading

Non-separable, recursive (dynamically consistent) preference structure.

$$U_t = \left[c_t^{1-\rho} + \beta E\left[U_{t+1}^{1-\gamma} \mid \mathcal{F}_t\right]^{\frac{1-\rho}{1-\gamma}}\right]$$

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Kreps–Porteus: preference for timing of information

- The above is a special case of the aggregator (after a transformation)

$$V_t = f\left(c_t, E\left[V_{t+1} \mid \mathcal{F}_t\right]\right)$$

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## Recall the concept of the no-trade theorem experiment

First open an ex-ante complete market where period $t$ consumption claims can be traded conditional on the history $\theta^t$.

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- Filtration $\{\mathcal{F}_t^*\}$.
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In a dynamic environment, we need to specify how we got to the initial Pareto optimal allocation.

### Two period example

1. Let the agents trade in a complete state-contingent market with information $\{\mathcal{F}_t\}$.

$$V_1 = \left[ c_1^{1-\rho} + \beta E \left[ c_2^{1-\gamma} \mid \mathcal{F}_1 \right]^{\frac{1-\rho}{1-\gamma}} \right]$$

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4. Now retrading can occur: Second round of trading is under different preferences.

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## INTERPRETATION 1: CHANGING THE PREFERENCE STRUCTURE

Notice that it is crucial that new information arrives as a surprise.

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- Dynamic inconsistency

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- Second round of trading under preference ranking $V_1^*$
- Dynamically consistent $\Longrightarrow$ no retrading.

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Under non-separable preferences, $x_t$ contracts matter.

- optimal time-$t$ consumption allocation is a function of the whole history

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Under non-separable preferences

- these two experiments are distinct
- the paper uses the incomplete market interpretation

1. Paper defines risk-aversion dominating preferences, which, $\forall C$,

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2. Why cannot we complete the markets to news signals $x_t$?

   - Agents would want to trade such contracts. What prevents it?

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- Many degrees of freedom that are hard to discipline.
- Quantification of 'news shocks' (*Barsky and Sims (2011), Sims (2012)*) that cannot be contracted upon ex ante.

Right now the quantitative model can generate large amount of retrading (volume).

- Proof of concept?
- Complete markets in payoff-relevant states.
- Perfect signal about next period state that is not contractible.

A more serious exercise should look at

- Precision of signals about the future (news shocks)
- Empirical evidence on (non)contractability of these shocks (derivative markets).