# Survey data and subjective beliefs in business cycle models

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**Abstract**

This paper develops a theory of subjective beliefs that departs from rational expectations, and shows that biases in household beliefs have quantitatively large effects on macroeconomic aggregates. The departures are formalized using model-consistent notions of *pessimism* and *optimism* which are supported by extensive time-series and cross-sectional evidence from household surveys. The role subjective beliefs play in aggregate fluctuations is quantified in a business cycle model with goods and labor market frictions. Consistent with the survey evidence, an increase in pessimism generates upward biases in unemployment and inflation forecasts and lowers economic activity. The underlying belief distortions reduce aggregate demand and propagate through frictional goods and labor markets. As a by-product of the analysis, solution techniques that preserve the effects of time-varying belief distortions in the class of linear solutions are developed.

*JEL classification:* E32, D84, E71

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# 1 Introduction

Survey data on households' expectations about future macroeconomic outcomes reveal significant systematic biases and comovement of these biases at business cycle frequencies. In this paper, we present a theory of subjective beliefs that departs from rational expectations and is disciplined using this survey evidence. Our theory formalizes these departures using model-consistent notions of *pessimism* and *optimism* and how they vary over the business cycle. Embedding this theory into a quantitative business cycle model, we show that fluctuations in the subjective belief biases drive a substantial share of movements in macroeconomic aggregates, particularly in the labor market.

We begin by documenting time-series and cross-sectional patterns in household forecasts for unemployment and inflation. Using the University of Michigan Surveys of Consumers, we show that household forecasts for unemployment and inflation are biased upward on average and both biases fluctuate significantly over the business cycle, increasing during recessions. Furthermore, in the cross section, households that forecast high inflation relative to the population also tend to forecast high unemployment. We also provide extensive micro-level evidence showing that biases in inflation and unemployment forecasts are tightly related to biases in forecasts of other aggregate and individual economic variables. These results are corroborated by additional evidence from the Survey of Consumer Expectations conducted by the Federal Reserve Bank of New York.

We then develop a framework that delivers these deviations of households' beliefs from their rational expectations counterpart as an outcome of time-varying pessimism or optimism. We model pessimism (optimism) as agents overweighting the probability of future states that deliver low (high) continuation utilities, and require dynamically consistent decision rules for agents acting under their subjective beliefs. Since continuation utilities depend on agents' actions and equilibrium prices, the framework endogenously determines the subjective beliefs jointly with macroeconomic aggregates, providing a set of overidentifying restrictions. The forecast biases that we measure in the data are identified by the difference between the subjective and rational expectations forecasts in the model. This mapping between the theory and survey data provides us with moment restrictions that we use for calibration.

We show that time-varying pessimism and optimism is an important source of macroeconomic risk by applying our framework to a calibrated economy with nominal rigidities and a frictional labor market. The rational expectations version of the model reproduces the well-known unemployment volatility puzzle. On the other hand, once we include belief biases that are calibrated to match the survey data, the model generates the empirically observed large volatility of labor market variables.

The mechanism through which fluctuations in beliefs affect the macroeconomy is consistent with the empirical evidence—an increase in pessimism is contractionary and increases the belief biases in both inflation and unemployment forecasts. Pessimism raises households' subjective probability of lower productivity growth, tighter monetary policy, and further increases in pessimism because these outcomes are associated with low continuation values. When consumers are more pessimistic, they lower current demand because of consumption smoothing. Monopolistically competitive intermediate goods firms expect lower future productivity and hence higher marginal costs, which reduces the incentive to lower prices. In the presence of labor market frictions, firms' pessimistic evaluation of future surpluses leads to lower match creation. In equilibrium, the decline in output and increase in unemployment due to an increase in pessimism is accompanied by a muted inflation response. Overall, agents are concerned about states with lower productivity, higher marginal costs, and tighter labor market conditions. This explains why our model generates countercyclical and positively correlated biases in inflation and unemployment forecasts.

Survey data provide an informative set of restrictions about the structure of the economy and sources of economic fluctuations. To illustrate these restrictions, we study two variants of the model. First, we consider a setting without TFP shocks. In this case, concerns about higher marginal costs are absent, and the model predicts a negative average inflation bias and a negative comovement between unemployment and inflation biases, both of which are counterfactual. The presence of uncertainty related to supply-type shocks is necessary to generate the correct sign and comovement of these biases.

Next, we study a variant with heterogeneous beliefs, in which we impose rational beliefs on the side of the firms. In this setting, an increase in pessimism consistent with the magnitude of fluctuations in unemployment biases still generates sizable responses of labor market variables, but the inflation bias is considerably attenuated compared with the benchmark. Firms with rational beliefs realize an increase in households' pessimism is contractionary, but similar to the case without TFP shocks, do not associate it with higher marginal costs, and inflation falls. Adverse states are therefore less correlated with high inflation, and pessimistic households overpredict inflation substantially less than in the data. The beliefs of firms therefore play an important role for the model to match the magnitude of the belief biases.

Our benchmark model features exogenous variation in pessimism. We relax this assumption by considering a setting in which increases in pessimism are triggered by negative TFP shocks. Overall, this specification produces fluctuations in macroeconomic aggregates and beliefs that match unconditional moments in the data as well as our benchmark model. However, dynamic responses to TFP shocks are counterfactually large, and the model fit to the empirical trajectories of the unemployment rate and subjective forecasts is weaker. This indicates a quantitatively important role for variation in pessimism that is orthogonal to productivity shocks.

We also present an alternative to the assumption of exogenous fluctuations in pessimism by considering a setting in which households face uninsurable idiosyncratic risk. In this setting, increases in idiosyncratic risk endogenously increase the belief biases without exogenous shocks to pessimism. We provide suggestive empirical evidence in favor of this mechanism by showing that our forecast biases are correlated with a proxy for idiosyncratic risk constructed by Schmidt (2016).

On the technical side, we develop a perturbation technique that incorporates the impact of time-varying belief biases in a first-order approximation of the model. The idea is to construct an appropriate scaling of the endogenously determined belief distortion that does not vanish as the perturbed economy approaches its deterministic limit. The approximation method leads to a tractable linear solution for the equilibrium dynamics with a role for subjective beliefs. The perturbation technique can be applied to a broader class of dynamic stochastic general equilibrium (DSGE) models, including settings in which agents have heterogeneous subjective beliefs. In our application, we use the heterogeneous belief setup to isolate the role of belief distortions of households and firms.

The paper contributes to the empirical and theoretical literatures that study deviations from full information rational expectations. A series of papers use household survey data to document empirical properties of forecast errors and test models of information frictions. For instance, see Carroll (2003), Mankiw et al. (2003), Coibion and Gorodnichenko (2012, 2015a), and Bordalo et al. (2020). In contrast to this literature, our focus is to build general equilibrium models disciplined by these survey data and study quantitative macroeconomic questions. In addition, our theory delivers a subjective measure for the *joint* distribution of outcomes and generates testable restrictions for the forecast errors across macroeconomic variables, which we confirm in the survey data.

Our model of pessimism and optimism is also related to a stream of literature that builds quantitative models of business cycles with information processing frictions (Mankiw and Reis (2007), Woodford (2013), Maćkowiak and Wiederholt (2015), Jurado (2016), Carroll et al. (2020)), extrapolative expectations (Eusepi and Preston (2011)), news and noise shocks (Beaudry and Portier (2004, 2007, 2014), Barsky and Sims (2011, 2012), Blanchard et al. (2013), Chahrour and Jurado (2018)), fluctuations in confidence (Angeletos et al. (2018)), fluctuations in discount rates (Hall (2017), Basu et al. (2021)), and model misspecification (Molavi (2019)). In contrast to these papers, we use household survey forecasts to discipline departures from rational expectations, in particular the average magnitude of the biases and their fluctuations, as well as their comovement across forecasts of different macroeconomic quantities.[1] These moments impose important restrictions on the capacity that belief biases have in generating fluctuations in macroeconomic fundamentals. We discuss differences between our approach and these theories in detail once we develop the belief model and its predictions in Section 3. We also show that the common component in fluctuations in the subjective belief biases, measured in the Michigan Survey, closely resembles several qualitative proxies for consumer confidence used in some of the above-mentioned work, with the benefit that survey data provide quantitative discipline on the magnitude of these belief biases. We elaborate on these connections in Section 6.

Our modeling of subjective beliefs utilizes the robust-preference framework developed by Hansen and Sargent (2001a,b), Strzalecki (2011), Hansen and Sargent (2016), and others. In this framework, agents act as if they faced a pessimistically biased probability distribution due to concerns about model misspecification. We instead take this pessimistically biased distribution as a model of subjective beliefs that we enrich to allow for time-variation in belief biases, and discipline the subjective beliefs using survey evidence. Our contribution to this literature is the parsimonious modeling of time-varying pessimism that can easily be applied to a large class of DSGE models

[1]A parallel literature studies empirical properties of survey forecasts on asset returns and embeds them in asset pricing models. See, for example, Bacchetta et al. (2009), Amromin and Sharpe (2014), Greenwood and Shleifer (2014), Barberis et al. (2015), Adam et al. (2017), Piazzesi et al. (2015), Adam and Merkel (2019), Szöke (2022), and Nagel and Xu (2022).

and the set of tools to compute and estimate equilibria with linear dynamics.

The robust-preference modeling foundations also imply connections of our work to quantitative models of ambiguity aversion in macroeconomics such as Bidder and Smith (2012), Ilut and Schneider (2014), and Bianchi et al. (2018). This literature used statistics like detection error probabilities or dispersion in analyst's forecasts as a way of calibrating the degree of model uncertainty. While we share with this literature the general notion of pessimistically slanted actions, we quantify the degree of pessimism directly using evidence from household survey answers, and use those moments to discipline the evolution of model-implied beliefs.

A recent and growing literature has followed approaches similar to ours in studying variation in pessimism and optimism in survey data and economic models. Kamdar (2018) uses a model of rational inattention to interpret survey evidence on household forecasts as driven by time variation in optimism and pessimism. Maenhout et al. (2021) and Szöke (2022) exploit survey evidence to discipline variation in pessimism in asset pricing models. Baqaee (2020) uses a model of ambiguity aversion to characterize asymmetries in inflation expectations in household survey data. Bassanin et al. (2021) construct a model of time-varying ambiguity concerns in a model of credit market fluctuations. Adam et al. (2021) analyze time-variation in pessimism and optimism in stock return forecasts. This recent work advocates that fluctuations in pessimism and optimism are a salient feature of the beliefs of economic agents and an important contributing factor to aggregate fluctuations.

The paper is organized as follows. Section 2 describes key empirical findings from the survey data. Motivated by these findings, we introduce our theory of subjective beliefs in Section 3, link the implications of the theory to the belief biases in survey data, and develop a tractable solution technique for approximating the equilibrium dynamics. Section 4 is devoted to the construction and calibration of the structural business cycle model that embeds the subjective belief model. In Section 5, we discuss implications of the findings and the role of subjective beliefs in business cycle dynamics. Section 6 provides further verification of the model mechanism using local-projection based dynamic responses and forecast error regressions, and relates belief distortions implied by our model to measures of confidence, sentiment, and disagreement used in the literature. Section 7 concludes. The appendix contains detailed derivations of the approximation method, description of the data, further empirical evidence and theoretical results, and robustness checks.

## 2 Survey expectations

We start by analyzing data on households' expectations from the University of Michigan Surveys of Consumers (Michigan Survey). This survey collects answers to questions about households' own economic situation as well as their forecasts about the future state of the economy. We document large upward biases in average forecasts of future inflation and unemployment. These biases vary systematically over the business cycle and across individual households in the cross section.[2]

[2]A detailed description of the construction of all data is provided in Appendix D.

We define a *belief wedge* as the deviation of a survey response from the corresponding rational expectations forecast. This requires taking a stand on how to determine the probability measure that generates the data.

Our preferred way is to use a vector autoregression (VAR) as a convenient and flexible way of describing the data generating measure. We follow the literature and consider several commonly used specifications to ensure that the characteristics of the belief wedges are not sensitive to the particular choice of the VAR. The details of the construction of the preferred specification and additional results for the alternative versions of the VAR are provided in Appendix D.3 and Appendix D.4, respectively.

As a robustness check, we also document patterns for the belief wedges constructed using responses in the Survey of Professional Forecasters (SPF) as the rational forecast. There is existing work that studies biases in SPF forecasts (Elliott et al. (2008), Capistrán and Timmermann (2009), Bordalo et al. (2020)), but these biases are substantially smaller than those we find in household surveys (Bianchi et al. (2022)) and are not robust to the chosen time period. Our main findings do not depend on these alternative choices of the rational forecast.

## 2.1 Time-series evidence

Figure 1 shows the differences between the Michigan Survey average household expectations and the rational forecasts for inflation and unemployment. The survey expectations are mean one-year-ahead expectations in the survey samples, constructed using quarterly data for the period 1982Q1–2019Q4. The unemployment rate survey forecast is inferred from categorical answers by fitting a time series of parametric distributions using the procedure from Carlson and Parkin (1975) and Mankiw et al. (2003). The construction of both series is detailed in Appendix D.1.

The belief wedges in Figure 1 are large on average, vary over time, and have a strong common component that is correlated with the business cycle. Using the VAR as the rational forecast, the average inflation and unemployment wedges over the sample period are 1.22% and 0.52%, respectively. The wedges are also volatile, with standard deviations of 0.97% and 0.57% for inflation and unemployment, respectively. Finally, the wedges consistently increase during the shaded NBER recessions. This means not only that households overestimate unemployment and inflation relative to the VAR forecast, but also that these biases are larger when measures of business activity are low. The correlations of the inflation and unemployment wedges with output gap are $-0.29$ and $-0.51$, respectively, and analogous correlations with GDP growth are $-0.49$ and $-0.28$, respectively.

The bottom panel of Figure 1 shows that these patterns are robust to using the SPF forecasts as the rational benchmark. In Figure 16 in Appendix D.4 we separately plot the household forecasts and the rational forecasts. This figure shows that these cyclical patterns in the belief wedges are driven not only by fluctuations in the VAR or SPF forecasts, but crucially by time variation in the actual household forecasts. In the same appendix, we also report the descriptive statistics using other ways of measuring the wedges, such as extending the data to a longer sample and using the median response across households.

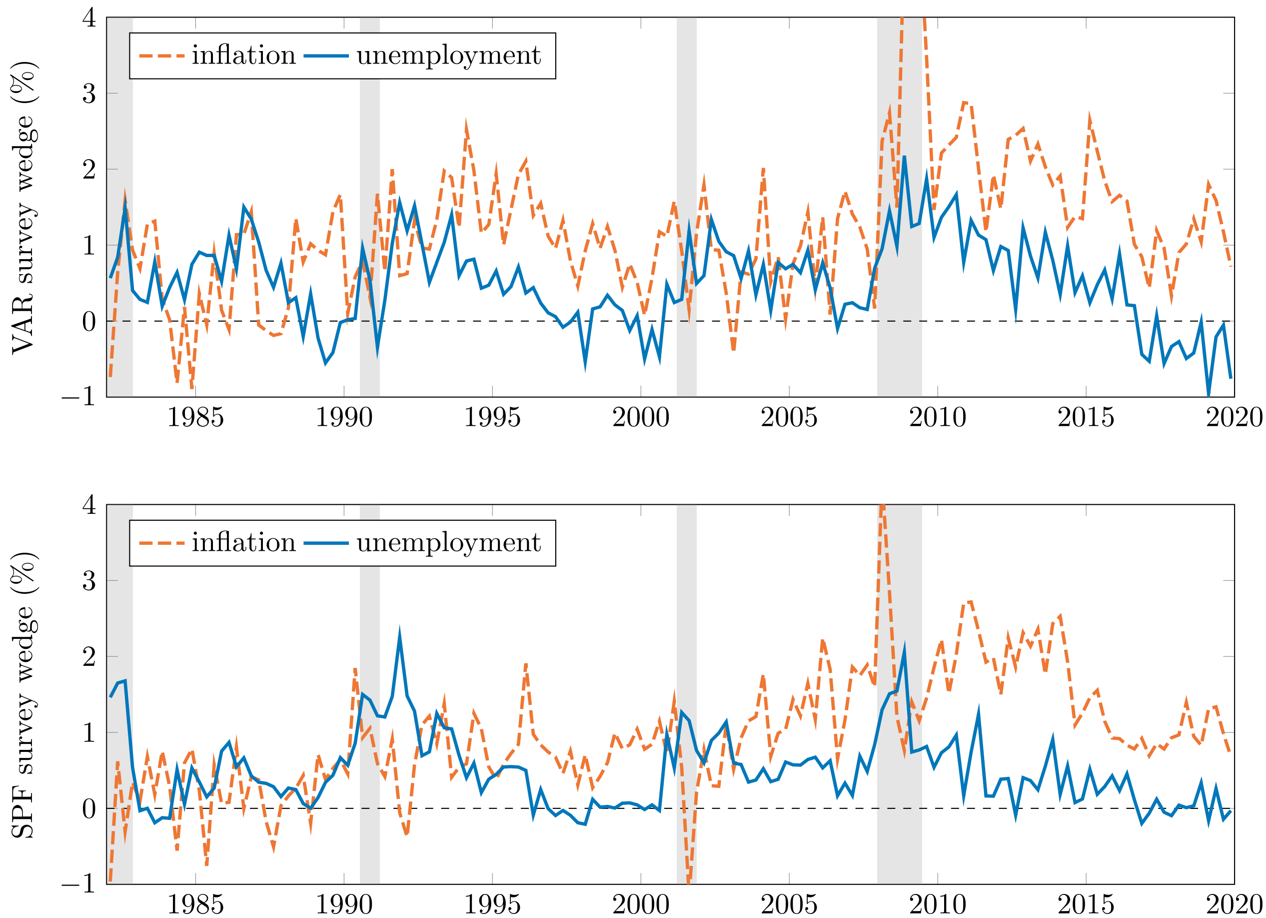


**Figure 1:** *Top panel:* Difference between the mean one-year-ahead forecasts from the Michigan Survey and corresponding statistical VAR forecasts. *Bottom panel:* Difference between the mean one-year-ahead forecasts from the Michigan Survey and corresponding SPF forecasts. Details on the construction of the data series are in Appendix D. NBER recessions are shaded.

We interpret these patterns as households expressing time-varying pessimism or optimism in their view of the aggregate economy. A pessimistic household overweighs the probability of adverse future states relative to the data-generating measure. Unemployment is high in these adverse states, and households' unemployment forecasts hence exhibit a positive belief wedge. Similarly, the observed positive inflation wedge implies that a pessimistic household views high inflation states as adverse.

The interpretation of positive inflation biases as emerging from a pessimistic view of the economy also lines up with survey evidence on households' inflation attitudes. The Bank of England administers a quarterly Inflation Attitudes Survey in which households are asked, among other questions, what the impact of an increase in inflation would be on the United Kingdom economy. Figure 2 shows that over the sample, between 50% and 80% of households responded that an increase in inflation would weaken the economy. Moreover, this fear of an adverse impact of higher inflation is highest during the Great Recession, and the correlation of this share of households with United Kingdom GDP growth over the 1999Q4–2019Q4 sample period is $-0.48$. The household

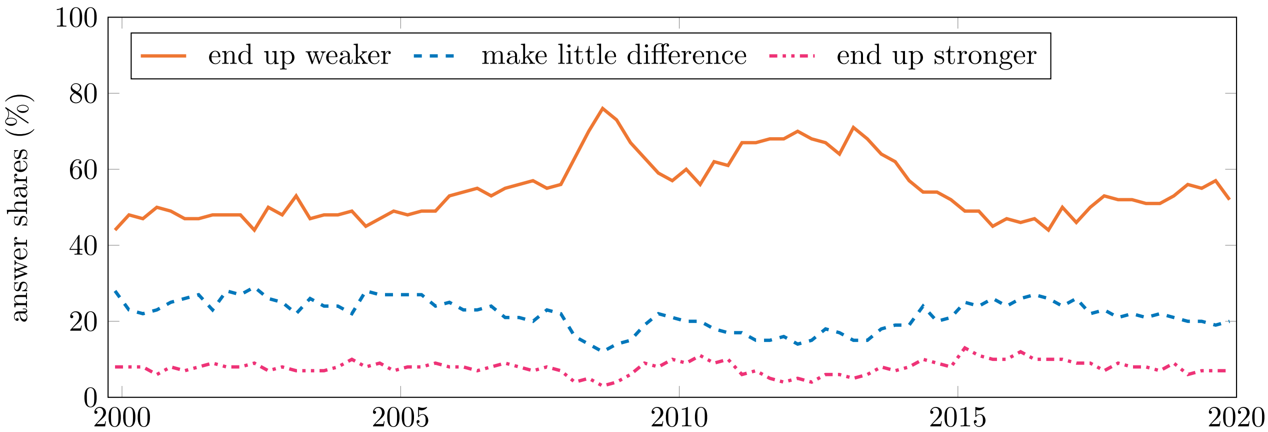


**Figure 2:** Bank of England Inflation Attitudes Survey, shares of answers to the question: "*If prices started to rise faster than they do now, do you think Britain's economy would ...*" Data sample 1999Q4–2019Q4.

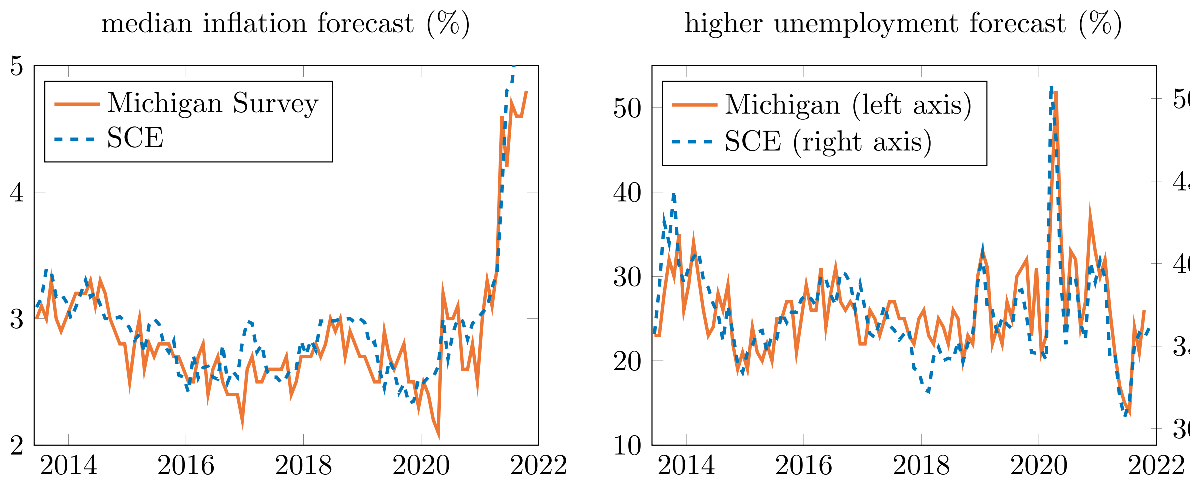


**Figure 3:** *Left panel:* Median inflation forecasts in the Michigan Survey and the SCE. *Right panel:* Share of respondents in the Michigan Survey stating that unemployment will be higher during the next 12 months and the mean probability that unemployment will be higher one year from now in the SCE. Details on the construction of the data series are in Appendix D.

median inflation forecast averaged over this sample is 2.74%, while the realized inflation rate over this period averaged 2.00%. Therefore, United Kingdom households significantly overpredict inflation, associate high inflation with adverse economic outcomes, and tend to have larger biases during recessions.[3] That households associate high inflation with adverse outcomes is also confirmed and discussed by Shiller (1997).

These patterns are robust to alternative ways of measuring the wedges. A particularly insightful check is a comparison of our results from the Michigan Survey with the Federal Reserve Bank of

[3]The large magnitude of the inflation wedge in household survey expectations is also consistent with the findings of Coibion and Gorodnichenko (2015b) for the United States, as well as with international evidence. For example, Coibion et al. (2018b) find large positive inflation biases in household and firm surveys in New Zealand and Vellekoop and Wiederholt (2019) document large and persistent positive biases in a long panel survey of households in the Netherlands.

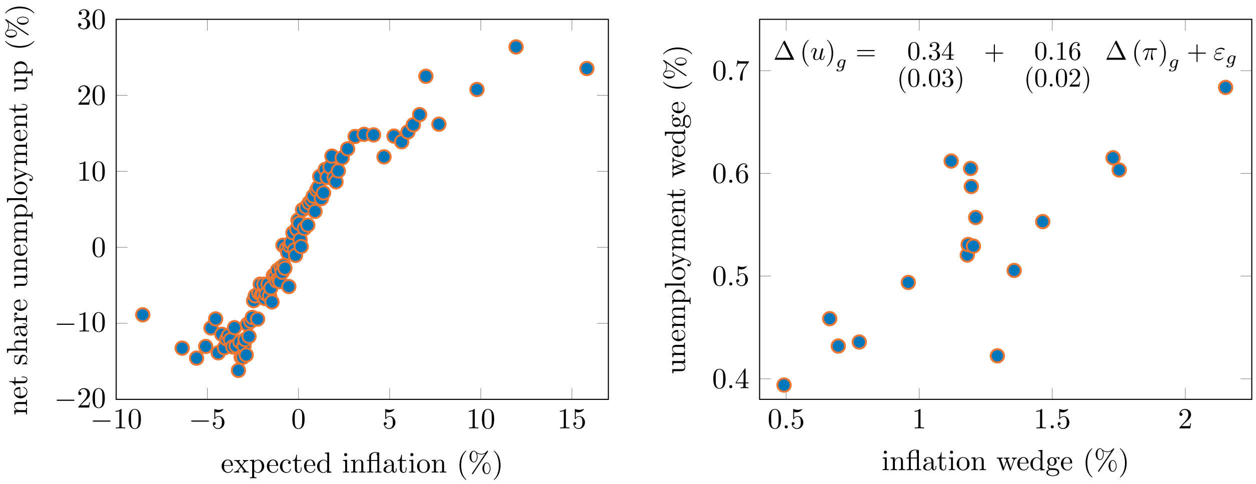


**Figure 4:** *Left panel:* Bin scatter plot for household-level data in the Michigan Survey. Households sorted into percentile bins on inflation forecasts, with time effects removed. Vertical axis represents the difference in shares of households forecasting unemployment going up and unemployment going down. *Right panel:* Relationship between the average inflation wedge $\Delta(\pi)_g$ of demographic group $g$ and the corresponding average unemployment wedge $\Delta(u)_g$ in the Michigan Survey. Demographic groups are listed in Table 7 in Appendix D. Standard errors of regression coefficients are in parentheses.

New York Survey of Consumer Expectations (SCE). The SCE contains a richer set of questions but only began in 2013. The left panel of Figure 3 shows that the median inflation forecasts from both surveys are very well aligned, including the timing and magnitude of the large increase in survey forecasts during the recent inflation wave.[4] Since the two surveys do not ask the same questions about unemployment, the right panel shows two different sets of unemployment forecast statistics. We report the mean probability that unemployment will be higher one year from now from the SCE and the share of respondents who predict that unemployment will be higher in the next 12 months from the Michigan Survey. The levels are not directly comparable but the statistics comove strongly over time.

## 2.2 Cross-sectional evidence

In addition to the time series, we also use household-level data to provide evidence for a positive cross-sectional correlation between the unemployment and inflation belief wedges and a strong comovement across time for disaggregated demographic groups. These patterns corroborate the idea that subjective beliefs about aggregate variables contain a common factor that reflects time-varying pessimism or optimism.

The left panel of Figure 4 shows the tight relationship between the inflation and unemployment forecasts at the micro level in the Michigan Survey. The scatter plot sorts the inflation forecasts of

[4] Armantier et al. (2013) and Manski (2017) advocate eliciting probabilistic forecasts from individual households. The Michigan Survey forecast is constructed by aggregating point forecasts of individual households, and we assume these to be the mean forecasts under the subjective distribution in the quantitative model. Households in the SCE report subjective distributions of the forecasted variables, which are then integrated to obtain mean forecasts at the household level. The alignment of the data from the two surveys justifies this assumption.

individual households into percentile bins, similarly to Candia et al. (2020), and plots the difference in the share of households in each bin that forecast unemployment going up and unemployment going down on the vertical axis. The plot reveals that households that forecast high inflation also predict high unemployment.

As we document in Appendix D.5, these cross-sectional patterns are prevalent for a range of survey responses. Households that expect high inflation also expect a general worsening of aggregate economic conditions measured using a variety of variables, as well as worsening of their own economic situation. Importantly, individual household forecasts also exhibit a tight relationship between forecasts of aggregate unemployment and probabilities of losing their own jobs. Moreover, all these patterns also hold when controlling for household-level fixed effects, implying that when households increase their inflation forecasts between subsequent interviews, they also predict more adverse economic conditions going forward. We corroborate these findings with data from the SCE, which shows analogous results using an additional range of forecasted quantities.

The cross-sectional heterogeneity in forecasts is persistent and related to demographic characteristics of the households. The right panel of Figure 4 displays evidence at the level of demographic groups reported in the Michigan Survey for average wedges over the examined period 1982Q1–2019Q4.[5] Demographic groups with larger average inflation wedges also have larger unemployment wedges. Consistent with existing evidence, households with lower reported education and lower reported income levels make more biased forecasts, but these biases remain nontrivial even for high-education and high-income households. These cross-sectional patterns are independent of the construction of the underlying rational forecast.[6]

We also plot the dispersion of the data from the Michigan Survey for the unemployment rate and inflation rate forecasts in Figure 5. For the inflation data, we have information on the quantiles of the cross-sectional distribution. For the unemployment rate forecast, we use the inferred distributions from categorical answers. The cross-sectional dispersion in the survey answers across individual households is substantial, but the interquartile range appears to be stable over time. The correlation between the mean and median inflation forecast is 0.94.

Overall, the time-series and cross-sectional evidence from the Michigan Survey paints a clear picture. Households on average expect higher unemployment and higher inflation relative to rational expectations, and these biases are larger in recessions.

## 3 Framework for subjective beliefs

Motivated by the empirical results from Section 2, we now introduce a framework for modeling deviations of agents' subjective beliefs from the data-generating probability measure. Denote the

[5]This demographic classification includes alternative age groups, geographical regions, quartiles of the income distribution, gender, and different levels of education. Table 7 in Appendix D provides additional details.

[6]In Appendix D.5, we show that this cross-sectional relationship is stable over time, holds at the level of individual households, and is robust to controlling for demographic composition. Rozsypal and Schlafmann (2019) and Das et al. (2020) also study these cross-sectional forecast patterns in the Michigan Survey and document results consistent with ours. Candia et al. (2020) provide further international evidence using household surveys from different countries.

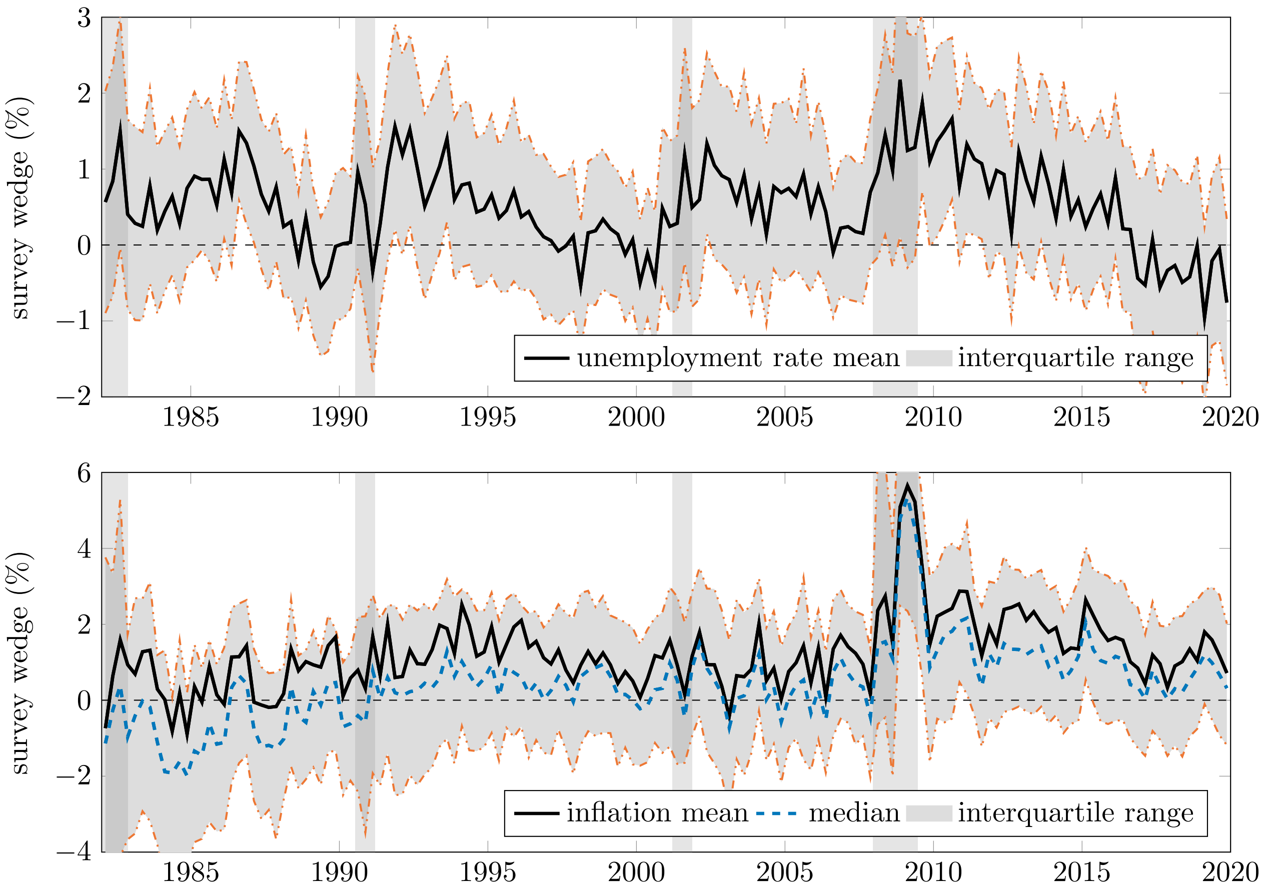


**Figure 5:** Dispersion in survey expectations in the Michigan Survey. The graphs show quantiles of the distribution of responses in the Michigan Survey, net of the mean VAR forecast. The top panel shows the unemployment responses, the bottom panel the inflation responses. Details on the construction of the data series are in Appendix D. NBER recessions are shaded.

data-generating and subjective probability measures as $P$ and $\widetilde{P}$, and the corresponding conditional expectations as $E_t[\cdot]$ and $\widetilde{E}_t[\cdot]$, respectively. The discrepancy between the two measures can be expressed using a strictly positive martingale with increments $m_{t+1}$.[7] The belief wedge for the one-period-ahead forecast of the macroeconomic variable $z_t$ is then given by

$$\Delta_t^{(1)}(z) \doteq \widetilde{E}_t[z_{t+1}] - E_t[z_{t+1}] = E_t[m_{t+1}z_{t+1}] - E_t[z_{t+1}]. \tag{1}$$

The random variable $m_{t+1}$, which captures agents' subjective beliefs, acts as a weighting scheme on the distribution of time-$t+1$ outcomes.

A model-consistent notion of pessimism (optimism) is a weighting scheme that overweights (underweights) states that are adverse from the agents' viewpoint. To formalize this idea in a

[7]Formally, under the assumption that $P$ and $\widetilde{P}$ are equivalent, there exists a strictly positive martingale $M$ defined recursively as $M_{t+1} = m_{t+1}M_t$ with $M_0 = 1$ such that for any $t$ and any time-$t+j$ measurable random variable $z_{t+j}$, $\widetilde{E}_t[z_{t+j}] = E_t[(M_{t+j}/M_t)\, z_{t+j}]$. The conditional distribution under $\widetilde{P}$ can therefore be fully characterized by specifying the process for $m_t$.

dynamically consistent environment, we build on the robust preference setting of Hansen and Sargent (2001a,b). Agents' preferences are represented using a concave period utility $u(\cdot)$ and the continuation value recursion

$$V_t = \min_{\substack{m_{t+1}>0 \\ E_t[m_{t+1}]=1}} u(x_t) + \beta E_t[m_{t+1}V_{t+1}] + \frac{\beta}{\theta_t} E_t[m_{t+1}\log m_{t+1}] \tag{2}$$

with

$$\theta_t = \overline{\theta} x_t, \tag{3}$$

$$x_{t+1} = \psi(x_t, w_{t+1}). \tag{4}$$

Here, $x_t$ is an $n \times 1$ vector of stationary economic variables that follows the Markovian law of motion (4), $\overline{\theta}$ is a $1 \times n$ vector of parameters, and $w_{t+1} \sim N(0_k, I_{k\times k})$ is an independent and identically distributed (iid) vector of normally distributed shocks under the data-generating probability measure $P$. We take the function $\psi$ as given for now but later derive it as a solution to a set of equilibrium conditions. The linear specification of $\theta_t$ allows for negative values, in which case the minimization problem in (2) turns into a maximization problem.[8]

When $\theta_t$ is positive, the minimization problem in (2) that characterizes the pessimistic belief can be interpreted as capturing agents' concerns about model misspecification. Each choice of a process $m_{t+1}$ yields an alternative model, i.e., an alternative probability distribution of future outcomes. The pessimistic agent is led to choose a model that yields a low expected value $E_t[m_{t+1}V_{t+1}]$ but wants to only consider as plausible models that are difficult to distinguish statistically from the data-generating measure. The degree of statistical similarity is controlled by the entropy penalty $E_t[m_{t+1}\log m_{t+1}]$, scaled by the penalty parameter $\theta_t$. More pronounced statistical deviations that are easier to detect are represented by random variables $m_{t+1}$ with a large dispersion that yields a large entropy. Optimal decisions and the pessimistic subjective beliefs that rationalize them are pinned down by the desire of the household to bound utility losses from potential model misspecification. In this paper, we refrain from interpreting the subjective belief as resulting from such model misspecification concerns. Instead, we postulate (2) directly as a specific model of pessimism or optimism, later disciplined using available survey data.

The solution to the minimization problem (2) satisfies

$$m_{t+1} = \frac{\exp(-\theta_t V_{t+1})}{E_t[\exp(-\theta_t V_{t+1})]}, \tag{5}$$

and $m_{t+1}$ completely characterizes agents' subjective beliefs relative to the data-generating measure. Adverse outcomes are states with low continuation values $V_{t+1}$. The sign of $\theta_t$ captures whether

[8] In Section 4, we also endow the agent with a set of controls, which gives rise to a max–min specification of the recursion. The multiplier version of the robust preference setup from Hansen and Sargent (2001a,b) is obtained when $\theta_t$ is a constant parameter. In Appendix C, we discuss the sequence formulation of the decision problem and link it to the robust control problem in more detail.

the agent is pessimistic or optimistic, and the magnitude of $\theta_t$ controls the magnitude of the belief distortion. An increase in $\theta_t$ corresponds to an increase in pessimism. The value $\theta_t = 0$ corresponds to $m_{t+1} = 1$, in which case the one-period-ahead subjective belief coincides with the data-generating process.[9]

Agents endowed with preference formulation (2) act as dynamically consistent subjective expected utility agents with beliefs given by the probability measure $\widetilde{P}$. Since $\widetilde{P}$ rationalizes their actions, we impose the hypothesis that agents answer survey questions about economic forecasts according to the same $\widetilde{P}$ and relate the belief wedges from Section 2 to the difference between expectations under $\widetilde{P}$ and the data-generating measure $P$.

Two observations motivate this hypothesis. First, as we documented in Section 2 and consistent with the large literature on household survey expectations, household survey data on economic forecasts exhibit substantial and persistent biases characterized by fluctuations in pessimism and optimism that (5) formally captures. Second, subjective beliefs reported in surveys are found to be systematically related to real consumption behavior. Similar to us, Malmendier and Nagel (2016) use the Michigan Survey to substantiate a significant relationship between survey responses on subjective expectations of economic outcomes and individual consumer spending, borrowing, and lending decisions. Ichiue and Nishiguchi (2015) use household survey data from Japan to link inflation expectations and durable goods spending. Vellekoop and Wiederholt (2019) link households' portfolio choice to their inflation expectations in Dutch survey data. Giglio et al. (2021) relate investors' portfolio choice and surveyed return expectations. Gennaioli et al. (2015) show that subjective expectations of managers in the Duke University CFO Survey have predictive power for firm investment and production behavior. Tanaka et al. (2020) document that subjective GDP forecasts of Japanese firms predict their employment, investment, and output growth. Lastly, Crump et al. (2022) exploit the SCE to estimate agents' intertemporal elasticity of substitution using the relationship between subjective inflation expectations and expected spending behavior. All these findings support the rationale for associating the survey answers with the subjective beliefs that households use in their decision making.

Combining equations (1) and (5) yields

$$\Delta_t^{(1)}(z) = Cov_t\left[m_{t+1}, z_{t+1}\right] = Cov_t\left[\frac{\exp\left(-\theta_t V_{t+1}\right)}{E_t\left[\exp\left(-\theta_t V_{t+1}\right)\right]}, z_{t+1}\right]. \tag{6}$$

The belief wedges associated with macroeconomic variables $z_{t+1}$ thus depend on their covariance with agents' continuation value $V_{t+1}$. In the context of the empirical evidence from Section 2, when $\theta_t > 0$ and agents are pessimistic, they overpredict unemployment *because* unemployment is high in states that they perceive as adverse. Since continuation values $V_{t+1} = V\left(x_{t+1}\right)$ and the law of

[9] In recent work, Caplin and Leahy (2019) use a specification with a negative $\theta_t$ to motivate optimistic beliefs, while Bassanin et al. (2021) specify a time-varying $\theta_t$ to model fluctuations in pessimism and optimism in a model of credit cycles. Substituting the solution for $m_{t+1}$ into problem (2) yields the recursion $V_t = u\left(x_t\right) - \frac{\beta}{\theta_t}\log E_t\left[\exp\left(-\theta_t V_{t+1}\right)\right]$. When the period utility function is logarithmic, this is mathematically equivalent to Epstein and Zin (1989) preference under unitary elasticity of substitution with a time-varying risk aversion coefficient $\gamma_t = \theta_t + 1$ used, for example, in Dew-Becker (2014), Alvarez and Atkeson (2017), or Basu et al. (2021).

motion (4) are endogenously determined, equation (6) combined with survey data yields a set of cross-equation restrictions for the equilibrium dynamics of the model.

### 3.1 General equilibrium and a solution method

We seek to incorporate the model of endogenous subjective beliefs into a large class of dynamic stochastic general equilibrium (DSGE) models. A wide range of DSGE models with subjective beliefs can be cast as a solution to a system of expectational difference equations,

$$0 = \widetilde{E}_t \left[ g \left( x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t \right) \right], \tag{7}$$

where $g_{t+1} = g \left( x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t \right)$ is an $n \times 1$ vector function.[10] This vector of equations includes agents' Euler equations, which can be represented using subjective beliefs implied by $m_{t+1}$. Specifically, for the $i$-th equation of the system,

$$0 = \widetilde{E}_t \left[ g^i_{t+1} \right] = E_t \left[ m_{t+1} g^i_{t+1} \right].$$

The feedback between agents' subjective beliefs and the equilibrium law of motion requires jointly solving the system of equations (7) for the continuation value recursion (2), the law of motion (4), and the endogenously determined probability measure $\widetilde{P}$ defined through (5).

We develop a novel approximation technique for the equilibrium dynamics of $x_t$ that builds on the series expansion method used in Borovička and Hansen (2014). The technique incorporates time variation in subjective beliefs in a tractable linear approximation of the equilibrium dynamics. Consider a class of models indexed by a perturbation parameter $\mathsf{q}$ that approximates the dynamics (4) by scaling the volatility of the innovations $w_{t+1}$:

$$x_{t+1} \left( \mathsf{q} \right) = \psi \left( x_t \left( \mathsf{q} \right), \mathsf{q} w_{t+1}, \mathsf{q} \right). \tag{8}$$

Hence, with each $\mathsf{q}$, there is an associated state vector process $x_t \left( \mathsf{q} \right)$ given by the law of motion (8), and $\mathsf{q} = 1$ recovers the original dynamics (4). The dynamics of $x_t \left( \mathsf{q} \right)$ are approximated by constructing a first-order series expansion,

$$x_t \left( \mathsf{q} \right) \approx \bar{x} + \mathsf{q} x_{1t}, \tag{9}$$

where the 'first-derivative' process $x_{1t}$ represents the local dynamics in the neighborhood of the steady state $\bar{x}$ and does not depend on $\mathsf{q}$. The steady state $\bar{x}$ is the solution to (8) evaluated at $\mathsf{q} = 0$, given implicitly by $\bar{x} = \psi \left( \bar{x}, 0, 0 \right)$. Assuming that the function $\psi \left( x, w, \mathsf{q} \right)$ is sufficiently smooth, we obtain the dynamics of $x_{1t}$ by differentiating (8) with respect to $\mathsf{q}$, utilizing (9), and

[10] Our solution method, fully described in Appendix B, is able to handle heterogeneous belief distortions for different forward-looking equations of the equilibrium system. We abstract from this heterogeneity in the main text to simplify notation but utilize this flexibility in Section 5 to disentangle the effect of belief distortions on the side of households and firms in our structural model.

evaluating at $\mathsf{q} = 0$:

$$x_{1t+1} = \psi_q + \psi_x x_{1t} + \psi_w w_{t+1}, \tag{10}$$

where $\psi_q$, $\psi_x$, and $\psi_w$ are conforming coefficient matrices representing the corresponding partial derivatives of $\psi(x, w, \mathsf{q})$ evaluated at the steady state. For example, $\psi_x \doteq \frac{\partial}{\partial x}\psi(x, w, \mathsf{q})\big|_{(\bar{x},0,0)}$.

The key innovation in our approach relative to the standard perturbation approximations in Sims (2002) or Schmitt-Grohé and Uribe (2004) is the treatment of the penalty parameter $\theta_t$ in the continuation value recursion when constructing the perturbation. Substituting the belief distortion (5) into (2) and applying the perturbation argument to the stochastic processes $V_t$, $x_t$, and $\theta_t$ yields the perturbed continuation value recursion

$$V_t(\mathsf{q}) = u(x_t(\mathsf{q}), \mathsf{q}) - \frac{\beta}{\theta_t(\mathsf{q})} \log E_t\left[\exp\left(-\theta_t(\mathsf{q}) V_{t+1}(\mathsf{q})\right)\right]. \tag{11}$$

The usual expansion in the perturbation parameter leads to the following first-order approximation of the exponent in (11) and in the numerator of (5):

$$-\theta_t(\mathsf{q}) V_{t+1}(\mathsf{q}) \approx -\bar{\theta}(\bar{x} + \mathsf{q}x_{1t})\left(\bar{V} + \mathsf{q}V_{1t+1}\right) \approx -\bar{\theta}\left(\bar{x}\bar{V} + \mathsf{q}\left(x_{1t}\bar{V} + \bar{x}V_{1t+1}\right)\right).$$

The scaling of the stochastic term by $\mathsf{q}$ indicates that as $\mathsf{q} \to 0$ (i.e., as the economy approaches its deterministic counterpart), the belief distortion in the perturbed model vanishes. Consequently, the usual first-order approximation of (11) is not affected by $\theta_t$, a standard result arising from the smoothness of the certainty-equivalent transformation $\log E_t[\exp(\cdot)]$.[11]

Instead, we propose to use the perturbation

$$\theta_t(\mathsf{q}) = \bar{\theta}x_t(\mathsf{q}) \approx \frac{\bar{\theta}(\bar{x} + x_{1t})}{\mathsf{q}}. \tag{12}$$

Differentiating (11) with respect to $\mathsf{q}$ then yields a recursion for the first-derivative process $V_{1t}$:

$$V_{1t} = u_x x_{1t} + u_q - \beta\frac{1}{\bar{\theta}(\bar{x} + x_{1t})} \log E_t\left[\exp\left(-\bar{\theta}(\bar{x} + x_{1t}) V_{1t+1}\right)\right]. \tag{13}$$

The nonlinearity in the recursive equation for the first-derivative process stems from the perturbation choice (12). Using the guess

$$V_{1t} = V_x x_{1t} + V_q, \tag{14}$$

recursion (13) yields a pair of equations for coefficients $V_x$ and $V_q$. The equation for $V_x$ is a Riccati equation whose solution can be found iteratively (see Appendix B.2). As a result, the zeroth-order approximation of the belief distortion (5) (i.e., the evaluation of the expansion of (5) at $\mathsf{q} = 0$) takes the form

$$m_{0t+1} = \frac{\exp\left(-\bar{\theta}(\bar{x} + x_{1t}) V_x \psi_w w_{t+1}\right)}{E_t\left[\exp\left(-\bar{\theta}(\bar{x} + x_{1t}) V_x \psi_w w_{t+1}\right)\right]}. \tag{15}$$

[11] The issue is analogous to the second-order nature of risk premia in small-noise approximations.

This expression reveals the effect of the perturbation choice (12). The volatility of the shocks $\mathsf{q}w_{t+1}$ in the perturbed economy (8) vanishes with $\mathsf{q} \to 0$, but at the same time, the magnitude of agents' belief biases (12) scales up relative to the shock volatility. These two effects are constructed to offset each other such that in the economy that approaches its deterministic limit, the agents' subjective model remains nontrivially distinct from the data-generating process. A similar assumption on the scaling of the perturbation parameter in the study of robustly optimal monetary policy is utilized in Adam and Woodford (2021).

When we approximate agents' subjective model $\widetilde{P}$ using the zeroth-order term of the belief distortion (15), the vector of normally distributed innovations $w_{t+1}$ in (4) under $\widetilde{P}$ has the distribution

$$w_{t+1} \sim N\left(-\overline{\theta}\left(\bar{x}+x_{1t}\right)\left(V_x \psi_w\right)', I_{k\times k}\right). \tag{16}$$

Instead of facing a vector of zero-mean shocks $w_{t+1}$, the agent perceives these shocks under her subjective beliefs as having a time-varying drift. The time variation is determined by the first-order dynamics of $\theta_t$ from equation (3), given by $\overline{\theta}\left(\bar{x}+x_{1t}\right)$. The relative magnitudes of the distortions of individual shocks are given by the sensitivity of the continuation value to the dynamics of the state vector, $V_x$, and the loadings of the state vector on individual shocks, $\psi_w$. An implication of (16) is that the dynamics of the model (10) under the agents' subjective beliefs $\widetilde{P}$ satisfy

$$\begin{aligned} x_{1t+1} &= \left[\psi_q - \psi_w \psi_w' V_x' \overline{\theta} \bar{x}\right] + \left[\psi_x - \psi_w \psi_w' V_x' \overline{\theta}\right] x_{1t} + \psi_w \widetilde{w}_{t+1} \\ &= \widetilde{\psi}_q + \widetilde{\psi}_x x_{1t} + \psi_w \widetilde{w}_{t+1}, \end{aligned} \tag{17}$$

where $\widetilde{w}_{t+1} \sim N\left(0_k, I_{k\times k}\right)$ is an iid vector of normally distributed shocks under the subjective probability measure $\widetilde{P}$.

### 3.2 Restrictions on subjective beliefs and mapping to survey data

Subjective beliefs alter both the conditional mean and the persistence of economic shocks. Moreover, variables that move $\theta_t$ and the continuation value in opposite directions exhibit a higher persistence under the subjective beliefs.[12] Assume that the forecasted variable $z_t$ in (1) takes the form $z_t = \bar{z}' x_t$. Using the linearized dynamics under the data-generating measure (10) and under the subjective measure (17), we obtain an expression for the model-implied belief wedges:

$$\begin{aligned} \Delta_t^{(1)}(z) &= \widetilde{E}_t\left[z_{t+1}\right] - E_t\left[z_{t+1}\right] = \bar{z}' \psi_w \widetilde{E}_t\left[w_{t+1}\right] \\ &= -\overline{\theta}\left(\bar{x}+x_{1t}\right) \bar{z}' \left(\psi_w \psi_w'\right) V_x'. \end{aligned} \tag{18}$$

Equation (18) is the linearized version of formula (6). Longer-horizon forecasts $\Delta_t^{(j)}(z) = \widetilde{E}_t\left[z_{t+j}\right] - E_t\left[z_{t+j}\right]$ are constructed correspondingly by iterating on the subjective dynamics (17).

The model predicts a one-factor structure in the dynamics of the belief wedges measured using the survey data. The relative distortions of survey forecasts of macroeconomic variables $z_t$ are given

[12] This statement is precisely correct in the scalar case when $\psi_x^2 V_x \overline{\theta} < 0$.

by constant loadings $-\bar{z}'(\psi_w\psi_w')V_x'$, whereas the factor that measures the overall magnitude of the belief distortions, $\theta_t \approx \overline{\theta}(\bar{x}+x_{1t})$, varies over time. This one-factor structure is the key restriction that the linearized subjective beliefs model imposes on the joint dynamics of the survey answers and implies that the magnitudes of the belief wedges should comove over time, which is consistent with the evidence in Section 2.[13]

The vector of loadings $-\bar{z}'(\psi_w\psi_w')V_x'$ is the negative of the covariance of the innovations to the value function, $V_x\psi_w$, with innovations $\bar{z}'\psi_w$ to the macroeconomic variable $z_t$ for which we have survey data. Since these loadings are determined endogenously in equilibrium, they also depend on the dynamics of the factor $\theta_t$, and hence on the vector $\overline{\theta}$. In the quantitative application in Section 4, we will study the implications of alternative parsimonious specifications of $\overline{\theta}$ for the relative magnitude of the belief distortions as well as for their time-series properties, and compare them with actual forecasts.

The model of beliefs therefore yields a set of overidentifying restrictions, both for the relative belief wedges across forecasted variables and for their time-series properties. Importantly, free parameters that characterize subjective beliefs are restricted to the specification of the process $\theta_t$ irrespective of the number of state variables or exogenous shocks. The number of overidentifying restrictions thus grows with the number of variables for which we have forecast data. While the theory itself does not predict the form of the factor $\overline{\theta}(\bar{x}+x_{1t})$ in (18), this common factor can be estimated as the common component in the time series data on belief wedges for alternative survey answers.

The structure of overidentifying restrictions also distinguishes the model implications from other mechanisms that affect aggregate fluctuations. In contrast to the extensive literature on various forms of Bayesian and non-Bayesian learning (Eusepi and Preston (2011)), information frictions (Mankiw and Reis (2007)), or news and noise shocks (Beaudry and Portier (2004)), this model of beliefs generates nonzero average biases that we confirm in the data. Second, as long as agents have correct beliefs, learning, information frictions, or news shocks cannot lead to forecast errors predictable by information available to those agents. Predictability of forecast errors is strongly supported by empirical evidence and existing literature that we discuss in Section 6.2.

The implications for the dynamics of macroeconomic quantities are more subtle. For example, in the case of news shocks, agents receive advance information about a future realization of a shock, which is subsequently confirmed in the data, contrary to the case of fluctuations in subjective beliefs. However, endogenous variables combine the impact of shocks and the consequences of the behavioral response to the change in beliefs, so distinguishing news shocks from pure belief fluctuations using endogenous quantities is possible but requires taking a stand on the specifics of the structure of the economy.

Using biases observed in survey data also helps us distinguish our model of fluctuations in

[13] The first-order expansion generates linear dynamics with homoskedastic shocks. In a fully nonlinear solution, time variation in the belief wedges $\Delta_t^{(j)}(z)$ would also incorporate fluctuations in the dispersion of $V_{t+1}$, generated, for example, by stochastic volatility. We leave a full examination of these considerations for future work but return to the role heteroskedastic shocks play in a specific example constructed in Section 5.4.

subjective beliefs from models of fluctuations in discount rates that generate variation in risk-free rates or rational risk premia (Hall (2017), Basu and Bundick (2017), Borovička and Borovičková (2019), Basu et al. (2021), Kehoe et al. (2023)). In Footnote 9, we note the equivalence between our model under logarithmic period utility and one with Epstein and Zin (1989) preferences and time-varying risk aversion. This equivalence implies that these two models cannot be distinguished using data on asset prices or macroeconomic variables. However, fluctuations in rational risk premia would not show up as biases in survey forecasts, allowing us to discipline the contribution of subjective beliefs themselves.

Models of ambiguity aversion (Bidder and Smith (2012), Ilut and Schneider (2014)) generate pessimistically slanted biases when the worst-case distribution is interpreted as a subjective belief. The previous literature calibrated the magnitude and time-variation of these distortions indirectly, by relying on measures like detection error probabilities and dispersion in analysts' forecasts. We differ from this literature by directly mapping the magnitudes of the biases to those observed in survey forecast data.

Brunnermeier and Parker (2005) formulate a model of optimal expectations in which subjective beliefs maximize average subjective utility under the correct ex-ante expectation. Similar to our setup with a negative $\theta_t$, subjective beliefs are jointly endogenously determined with the utility consequences of agents' decisions, and overweigh favorable states that are still plausible under the data-generating measure. Unlike the beliefs in our model, the Brunnermeier and Parker (2005) optimal expectations are determined ex-ante and not as an outcome of a dynamically consistent problem. Furthermore, the survey data we use predominantly favor pessimistic biases.

The endogenous determination of agents' beliefs implies that agents fear states that are adverse from their own perspective. In an environment with heterogeneous agents, this will also lead to belief heterogeneity associated with exposure to uninsurable idiosyncratic risk or life-cycle patterns, yielding testable restrictions that can be verified against the cross-sectional patterns in belief biases documented in Appendix D.5.

Finally, the first-order expansion generates linear dynamics with homoskedastic shocks. In a fully nonlinear solution, time variation in the belief wedges $\Delta_t^{(j)}(z)$ in (18) would also incorporate fluctuations in the dispersion of $V_{t+1}$, generated, for example, by stochastic volatility. The model then predicts that more volatile times that lead to more dispersion in $V_{t+1}$ also imply larger belief biases, providing a potential rationale for the cyclical behavior of biases we observe in the data.

We leave a full examination of heteroskedastic shocks, idiosyncratic risk, and implied belief heterogeneity for future work but return to the role heteroskedastic shocks and idiosyncratic risk play in a specific example constructed in Section 5.4.

# 4 Subjective beliefs in a structural business cycle model

In this section, we introduce the subjective beliefs framework into a calibrated version of an economy with nominal rigidities and a frictional labor market. In the absence of belief distortions, our environment is similar to Ravenna and Walsh (2008), Gertler et al. (2008), and Christiano et al. (2016). Our setup provides well-defined notions of unemployment and inflation, which directly map to the survey questions. Although parsimonious, the model is able to match moments of both the belief wedges and macroeconomic aggregates. We use this model to quantify the contribution of fluctuations in subjective beliefs to macroeconomic outcomes and assess key channels in the propagation mechanism.

## 4.1 Model

The model economy is populated by a representative household with subjective beliefs described in Section 3, competitive producers of a homogeneous final good, and monopolistic producers of intermediate goods who employ workers hired in a frictional labor market. In the benchmark version of the model, all economic agents share the same subjective beliefs as the representative household. Alternative specifications that distinguish between the beliefs of households and firms are studied in Section 5.[14]

### 4.1.1 Representative household

The preferences of the representative household are given by the recursion

$$V_t = \max_{C_t, B_{t+1}} \min_{\substack{m_{t+1}>0 \\ E_t[m_{t+1}]=1}} \log C_t + \beta E_t \left[ m_{t+1} V_{t+1} \right] + \frac{\beta}{\theta_t} E_t \left[ m_{t+1} \log m_{t+1} \right], \tag{19}$$

with time preference coefficient $\beta$. We impose a simple structure on the penalty parameter $\theta_t$ from equation (3), and assume that it follows an exogenously specified AR(1) process[15]

$$\theta_t = (1-\rho_\theta)\mu_\theta + \rho_\theta \theta_{t-1} + \sigma_\theta w_t^\theta. \tag{20}$$

We will refer to $\theta_t$ as the *belief shock*. The magnitude of the belief distortion is determined by fluctuations in $\theta_t$ specified in (20). However, equilibrium dynamics in the model endogenously determine the states that yield low continuation values $V_{t+1}$. These states are evaluated as adverse by the household and are then perceived as more likely under the subjective model.

Naturally, the dynamics of the subjective beliefs then endogenously depend on the structure of other shocks in the model, which we describe in Section 4.1.4. Households understand that endogenous dynamics generated by fluctuations in $\theta_t$ constitutes a source of risk, and take this into account when forming their subjective belief. In Section 5.3, we also study a version of the model

[14]The subjective belief of the representative household is calibrated to the survey evidence from the Michigan Survey. We abstract from explicitly modeling belief heterogeneity within the household sector, including households who could possibly be endowed with beliefs corresponding to the SPF forecasters. As in Section 2, the SPF forecasts only serve as one way of quantifying the belief biases of the representative household for the purpose of calibrating the model.

[15]This is a slight abuse of notation relative to equation (3) where $\theta_t = \bar{\theta} x_t$. Here, $\theta_t$ can be interpreted as one element of the state vector $x_t$, and $\bar{\theta}$ as a coordinate vector that selects this component.

in which we replace (20) with a process for $\theta_t$ that is correlated with the TFP process, which will alter the implications for relative magnitudes and dynamics of the wedges.

The household consists of a unit mass of workers who perfectly share consumption risk. A fraction $L_t$ is employed and earns a real wage $\xi_t$. A fraction $1-L_t$ is unemployed and collects unemployment benefits with real value $D$ financed through lump-sum taxes. The household faces the nominal budget constraint

$$P_tC_t + B_{t+1} \leq (1-L_t)\,P_tD + L_tP_t\xi_t + R_{t-1}B_t - T_t,$$

where $P_t$ is the price of consumption goods, $B_{t+1}$ denotes the one-period risk-free bonds purchased in period $t$ with return $R_t$, and $T_t$ are lump-sum taxes net of profits.

#### 4.1.2 Labor market

At the end of period $t-1$, employed workers separate with probability $1-\rho$ and join the pool of unemployed, who search for jobs at the beginning of period $t$. The total number of searchers at the beginning of period $t$ therefore is $1-\rho L_{t-1}$. The law of motion for the mass of employed workers is given by

$$L_t = \rho L_{t-1} + (1-\rho L_{t-1})\,f_t = (\rho + h_t)\,L_{t-1},$$

where $f_t$ is the endogenously determined job-finding probability and

$$h_t = \frac{f_t\,(1-\rho L_{t-1})}{L_{t-1}}$$

is the hiring rate. Measured unemployment is given by $u_t = 1-L_t$, which includes people who do not rejoin employment after searching at the beginning of the period.

Firms in the labor market hire workers and sell labor services using a linear technology. At the beginning of period $t$, they post vacancies at rate $v_t$ for a total number of vacancies $v_tL_{t-1}$. The labor market tightness is defined as the number of posted vacancies over the number of searchers,

$$\zeta_t = \frac{v_tL_{t-1}}{1-\rho L_{t-1}}.$$

A Cobb–Douglas matching function with efficiency $\mu$ and curvature $\nu$ combines vacancies and workers to produce

$$M_t = \mu\,(v_tL_{t-1})^{\nu}\,(1-\rho L_{t-1})^{1-\nu}$$

matches. The probability that a searching worker finds a job is then given by

$$f_t = \frac{M_t}{1-\rho L_{t-1}} = \mu\zeta_t^{\nu},$$

and the vacancy-filling rate is equal to $q_t = f_t/\zeta_t$.

We now characterize workers' subjective valuations when they are employed and unemployed, and the subjective value of the firm in the labor market. Workers' idiosyncratic risk from matching is pooled at the level of the household but workers are risk averse with respect to aggregate shocks faced by the household. Let $s_{t+1} = \beta C_t / C_{t+1}$ denote the marginal rate of substitution of the representative household between consumption today and consumption tomorrow. The value of an unemployed worker $U_t$ is given recursively as

$$U_t = D + \widetilde{E}_t \left[ s_{t+1} \left( f_{t+1} J^w_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right],$$

where $\widetilde{E}_t [\cdot]$ represents the expectation under the subjective belief of the household, and $J^w_t$ is the value of an employed worker. Similarly, the value of the employed worker satisfies the recursion

$$J^w_t = \xi_t + \widetilde{E}_t \left[ s_{t+1} \left( \rho + (1 - \rho) f_{t+1} \right) J^w_{t+1} \right] + \widetilde{E}_t \left[ s_{t+1} (1 - \rho) (1 - f_{t+1}) U_{t+1} \right].$$

Here, the term $\rho + (1 - \rho) f_{t+1}$ combines the probability $\rho$ of continuing in the existing job and the probability $(1 - \rho) f_{t+1}$ of losing the job at the end of period $t$ but immediately finding a new job at the beginning of period $t + 1$. Finally, the value of the worker to the firm is the present value of profits earned by the firm from the match, given by the difference between the worker's marginal product $\vartheta_t$ on the current job and the wage,

$$J_t = \vartheta_t - \xi_t + \rho \widetilde{E}_t \left[ s_{t+1} J_{t+1} \right].$$

To close the labor market, we specify the free-entry condition and the wage-setting protocol. Let $\kappa_v$ be the flow cost of posting a vacancy. The zero-profit condition for entering firms implies

$$J_t = \frac{\kappa_v}{q_t}.$$

We follow Shimer (2010) and use Nash bargaining with rigid wages. The firm and the worker bargain over a target wage $\xi^*_t$ to split the match surplus according to

$$\eta \left( J_t + \xi_t - \xi^*_t \right) = (1 - \eta) \left( J^w_t - U_t + \xi^*_t - \xi_t \right),$$

where $\eta$ is the bargaining power of the worker. The terms $J_t + \xi_t - \xi^*_t$ and $J^w_t - U_t + \xi^*_t - \xi_t$ are the surplus values to the firm and worker, respectively, of choosing the target wage $\xi^*_t$ instead of the equilibrium wage $\xi_t$. The actual wage is a weighted average of last period's wage and the current target wage,

$$\xi_t = \lambda \xi_{t-1} + (1 - \lambda) \xi^*_t,$$

where $\lambda$ is a wage rigidity parameter, with $\lambda = 0$ corresponding to flexible wages.

An important feature of the frictional labor market is the forward-looking nature of vacancy-posting decisions and bargaining. When evaluating the distribution of future states, workers inherit the beliefs of the representative household. Similarly, firms maximize profits using equilibrium state prices obtained from households' preferences and beliefs. This implies that fluctuations in $\theta_t$ directly affect the incentives of firms to post vacancies, through their effect on the valuation of the match surplus and equilibrium wages. This is a striking difference relative to the Walrasian spot market where workers are hired using only one-period employment contracts. In such an environment, fluctuations in subjective beliefs do not directly affect labor market decisions, since there is no uncertainty about economic conditions in the current period.

### 4.1.3 Production and market clearing

The frictional labor market is embedded in a New Keynesian framework with Calvo (1983) price setting. A homogeneous final good $Y_t$ with price $P_t$ is produced in a competitive market using the production technology

$$Y_t = \left[ \int_0^1 (Y_{j,t})^{(\varepsilon-1)/\varepsilon} \, dj \right]^{\varepsilon/(\varepsilon-1)}, \qquad \varepsilon > 0,$$

where $Y_{j,t}$ are specialized inputs with prices $P_{j,t}$. Final good producers solve the static competitive problem

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj,$$

leading to the first-order conditions

$$Y_{j,t} = \left( \frac{P_t}{P_{j,t}} \right)^{\varepsilon} Y_t, \qquad j \in [0,1] \, .$$

Specialized inputs are produced by monopolistic retailers indexed by $j$, using the technology

$$Y_{j,t} = \exp(a_t) \, l_{j,t} - \phi,$$

where $l_{j,t}$ is the quantity of labor services used in production, $a_t$ is the logarithm of the neutral technology level, and $\phi$ is a fixed cost of production. Labor services are purchased from labor market firms described in Section 4.1.2 at a competitive price $\vartheta_t$. The retailer is subject to Calvo-style price frictions and reoptimizes the price with probability $1 - \chi$. These infrequent adjustments imply that price setting is a dynamic problem affected by distortions in the firm's beliefs.

Aggregate resources satisfy the constraint

$$C_t + \frac{\kappa_v}{q_t} h_t L_{t-1} = Y_t$$

and the market clearing condition for labor services is

$$\int_0^1 l_{j,t} dj = L_t.$$

### 4.1.4 Shock structure and monetary policy

We complete the model by specifying the remaining sources of exogenous variation to the economy. The monetary authority follows an interest rate rule

$$\log\left(R_t/\overline{R}\right) = \rho_r \log\left(R_{t-1}/\overline{R}\right) + (1-\rho_r)\left[r_\pi \log\left(\pi_t/\overline{\pi}\right) + r_y \log\left(Y_t/Y^*\right)\right] + \sigma_r w_t^r,$$

where $w_t^R$ is an iid monetary policy shock, $\overline{\pi}$ an inflation target, and $Y^*$ the steady-state value of $Y_t$. The neutral technology process $a_t$ is specified as

$$a_{t+1} = \rho_a a_t + \sigma_a w_{t+1}^a.$$

The final source of exogenous variation is the belief shock $\theta_t$ specified in (20) that drives agents' subjective belief deviations from the data-generating process. We assume that all innovations are independent under the data-generating measure $P$:

$$\left(w_t^r, w_t^a, w_t^\theta\right)' \overset{iid}{\sim} N\left(0, I\right).$$

As we have seen in Section 3, this property does not carry over to the subjective measure where the joint distribution of future realizations of the innovations depends on the current level of $\theta_t$.

## 4.2 Model solution and calibration

The equilibrium of the structural model from the previous section fits in the general framework that we developed in Section 3.[16] We apply the expansion methods from Section 3.1 to compute a linear approximation to the solution for the equilibrium dynamics. Most parameters are calibrated to discipline the steady state of the economy and its dynamic responses to technology and monetary policy shocks. Parameters for the TFP process are estimated using data from Fernald (2014), and the parameters of the process $\theta_t$ are set to make the model-implied belief wedges for inflation and unemployment consistent with the data from Section 2. The calibrated parameter values are summarized in Table 1.

**Steady state.** The subjective discount factor $\beta = 0.994$ is set to target a steady state real return of 1% per year, and the intercept of the monetary policy rule $\bar{\pi} = 0.01$ to yield a steady-state annualized inflation of 2%.[17] The parameters $\varepsilon = 6$ and $\chi = 0.75$ governing nominal frictions are calibrated to match a markup of 20% and a frequency of price changes that corresponds to three quarters. We choose the job survival parameter $\rho = 0.89$ to target a quarterly separation rate of 11%. We follow Shimer (2005) and set the curvature of the matching function $\nu = 0.72$ and the worker's bargaining weight $\eta = 0.72$. The remaining labor market parameters—matching efficiency

[16] The full set of equilibrium equations of the model is stated in Appendix I.

[17] The steady state of the linearized model is distorted by first-order effects of belief distortions captured by the term $\psi_q$ in equation (10), so that the steady-state real return and inflation differ from $1/\beta - 1$ and $\bar{\pi}$, respectively.

| Parameters | | Value |
|---|---|---|
| $\beta$ | Discount factor | 0.994 |
| $\varepsilon$ | Elasticity of substitution intermediate goods | 6.00 |
| $\chi_p$ | Calvo price stickiness | 0.75 |
| $\chi_w$ | Wage rigidity | 0.925 |
| $\lambda$ | Steady state markup | 1.2 |
| $\bar{\pi}$ | Monetary policy rule: intercept | 0.01 |
| $\rho_r$ | Monetary policy rule: smoothing | 0.84 |
| $r_\pi$ | Monetary policy rule: loading on inflation | 1.60 |
| $r_y$ | Monetary policy rule: loading on output | 0.028 |
| | **Labor market** | |
| $\rho$ | Job survival probability | 0.89 |
| $\mu$ | Matching efficiency | 0.67 |
| $\nu$ | Curvature of matching function | 0.72 |
| $\eta$ | Worker's bargaining weight | 0.72 |
| $\kappa_v$ | Vacancy posting costs | 0.09 |
| $D$ | Flow benefits of unemployment | 0.57 |
| | **Shocks** | |
| $\mu_\theta$ | Mean belief shock | 5.64 |
| $\rho_\theta$ | Persistence of belief shock | 0.714 |
| $\sigma_\theta$ | Volatility of belief shock | 4.3 |
| $\rho_a$ | Persistence of TFP shock | 0.84 |
| $100\sigma_a$ | Volatility of TFP shock | 0.568 |
| $100\sigma_r$ | Volatility of monetary policy shock | 0.078 |

**Table 1:** Benchmark parameter values. Model is calibrated at a quarterly frequency.

$\mu = 0.67$, unemployment benefits $D = 0.57$, and vacancy posting costs $\kappa = 0.09$—are calibrated to achieve an average job-finding rate of 0.67, a flow value of unemployment equal to 70% of wages, and a steady-state tightness normalized to one.

**TFP and monetary policy shocks.** As shown in equation (18), belief distortions in the model depend on the size and propagation of fundamental shocks. We therefore discipline the calibration by aligning the dynamic responses to TFP and monetary policy shocks with their empirical counterparts. The model then endogenously determines how households distort these empirically relevant shocks to generate belief wedges, which we compare with the data as a check of model fit.

We use TFP data from Fernald (2014) to infer $\rho_a = 0.841$ and $\sigma_a = 0.568\%$. The smoothing parameter in the monetary policy rule $\rho_r = 0.84$ is taken from Christiano et al. (2016). The wage rigidity parameter $\lambda = 0.925$ and the monetary policy parameters $r_\pi = 1.60$, $r_y = 0.028$, and $\sigma_r = 0.078\%$ are chosen to make the impulse responses of inflation and unemployment to TFP and monetary policy shocks consistent with the VAR evidence in Dupor et al. (2009) and Christiano et al. (2016). As shown in equation (18), belief distortions in the model depend on the propagation of fundamental shocks, and it is therefore crucial for the model to fit these responses. We focus on cumulative 10-quarter responses as a target. The model generates cumulative inflation responses of $-2.79\%$ (TFP) and $-0.73\%$ (monetary policy) and cumulative unemployment responses of $-0.21\%$ (TFP) and 0.74% (monetary policy), which are values that fall comfortably within the estimated 90% confidence intervals in both papers.

**Belief shock.** Finally, we calibrate the parameters of the process $\theta_t$ using the belief wedge data from Section 2. Our model predicts a one-factor structure of the belief wedges (18). A straightforward procedure to extract a common factor is to consider the first principal component. In our data, the first principal component explains 76% of the variation in the wedges.[18] We set the autocorrelation coefficient $\rho_\theta$ to 0.714, which matches the autocorrelation of the first principal component extracted from the time series of unemployment and inflation wedges. The remaining two parameters for the mean and volatility of the belief shock, $\mu_\theta = 5.64$ and $\sigma_\theta = 4.3$, are chosen to fit four moments, namely the means and volatilities of the unemployment and inflation wedges.

### 4.3 Model fit

The first two columns of Table 2 show the fit of the calibrated benchmark model to moments for inflation, unemployment, output, and the belief wedges. The model somewhat understates the mean and volatility of the inflation wedge and the volatility of unemployment but matches the remaining moments of macroeconomic variables and belief wedges well. The model also matches meaningfully well the correlations of belief wedges and unemployment with output.

What the model misses is the unconditional acyclicality of inflation observed in the data. In the DSGE literature that studies models with a richer structure (e.g., Smets and Wouters (2007)), much of the variation in inflation is attributed to wage and price markup shocks which, at the same time, have little explanatory power for output and unemployment. Adding such shocks would reduce the correlation between inflation and output while having little impact on the dynamics of the wedges.

In the absence of such shocks, we focus on fitting the conditional responses of inflation to the TFP shock and belief shock, which determine the dynamics of the inflation wedge. We used the magnitude of responses to identified TFP shocks as targeted moments in the calibration of the model in Section 4.2. Later in Section 6.1, we compare the model-implied responses to the belief shock with empirical responses obtained using local projections. In Appendix E.1, we provide a more extensive discussion of the fit of the benchmark model in terms of its unconditional moments that yields additional insights regarding the model fit and the contribution of alternative shocks to the dynamics of the model.

We also consider versions of the model with alternative configurations of the shock processes and

[18] We also estimated the time series for the factor using a hidden factor model where we interpret the imperfect correlation between the two wedges as arising from measurement errors in the survey data. In Appendix F.1, we compare both estimates of the common factor. Their correlation in the time series is 0.95.

| Moment | Data | Model benchmark | no $\theta_t$ | only $\theta_t$ |
|---|---|---|---|---|
| Mean of inflation wedge | 1.22 | 0.90 | 0.00 | 0.00 |
| Mean of unemployment wedge | 0.52 | 0.55 | 0.00 | 0.00 |
| Volatility of inflation wedge | 0.97 | 0.73 | 0.00 | 0.00 |
| Volatility of unemployment wedge | 0.57 | 0.45 | 0.00 | 0.00 |
| Volatility of inflation | 1.37 | 1.17 | 0.98 | 0.00 |
| Volatility of output | 2.00 | 2.22 | 1.55 | 0.00 |
| Volatility of unemployment | 1.70 | 1.39 | 0.55 | 0.00 |
| Corr(inflation wedge, output) | −0.30 | −0.67 | 0.00 | 0.00 |
| Corr(unemployment wedge, output) | −0.49 | −0.67 | 0.00 | 0.00 |
| Corr(inflation, output) | 0.10 | −0.82 | −0.85 | 0.00 |
| Corr(unemployment, output) | −0.87 | −0.74 | −0.33 | 0.00 |

| Moment | Model no $a_t$ | $\theta(a_t)$ | flexible $w$ | rational firms |
|---|---|---|---|---|
| Mean of inflation wedge | −0.32 | 0.85 | 0.34 | 0.35 |
| Mean of unemployment wedge | 0.54 | 0.56 | 0.16 | 0.55 |
| Volatility of inflation wedge | 0.26 | 0.68 | 0.28 | 0.29 |
| Volatility of unemployment wedge | 0.43 | 0.46 | 0.13 | 0.45 |
| Volatility of inflation | 0.69 | 1.47 | 0.86 | 1.09 |
| Volatility of output | 0.98 | 3.14 | 1.49 | 1.98 |
| Volatility of unemployment | 0.87 | 1.49 | 0.77 | 1.24 |
| Corr(inflation wedge, output) | 0.50 | −0.98 | −0.44 | −0.53 |
| Corr(unemployment wedge, output) | −0.50 | −0.98 | −0.44 | −0.53 |
| Corr(inflation, output) | 0.17 | −0.79 | −0.64 | −0.53 |
| Corr(unemployment, output) | −0.98 | −0.99 | −0.35 | −0.67 |

**Table 2:** Data and model-implied theoretical moments for macroeconomic quantities and belief wedges. The sample period for the Data column is 1982Q1–2019Q4. Values in all columns are in percentages and annualized, output is detrended, inflation rate is the 4-quarter change in the price index.

model parameters to understand the role of fluctuations in $\theta_t$ and model specification in generating these results. We report the fit of these alternatives in the remaining five columns of Table 2.

To highlight the role of belief shocks in matching the unemployment volatility observed in the data, the third column (no $\theta_t$) reports statistics for the rational expectations version of the model ($\mu_\theta = \sigma_\theta = 0$). In this case, the model generates the standard unemployment volatility puzzle, with TFP and monetary policy shocks able to generate only a third of the empirically observed unemployment volatility. Belief wedges are zero by construction.

Figure 6 further emphasizes the importance of the $\theta_t$ shocks by comparing the model-implied paths for unemployment and belief wedges against the data. We extract the innovations to the TFP process from the Fernald (2014) time series and innovations to the belief shock process from the time series for the first principal component of the belief wedges that we used in the calibration, and feed them into the model. We compare the model-implied path for unemployment to a counterfactual

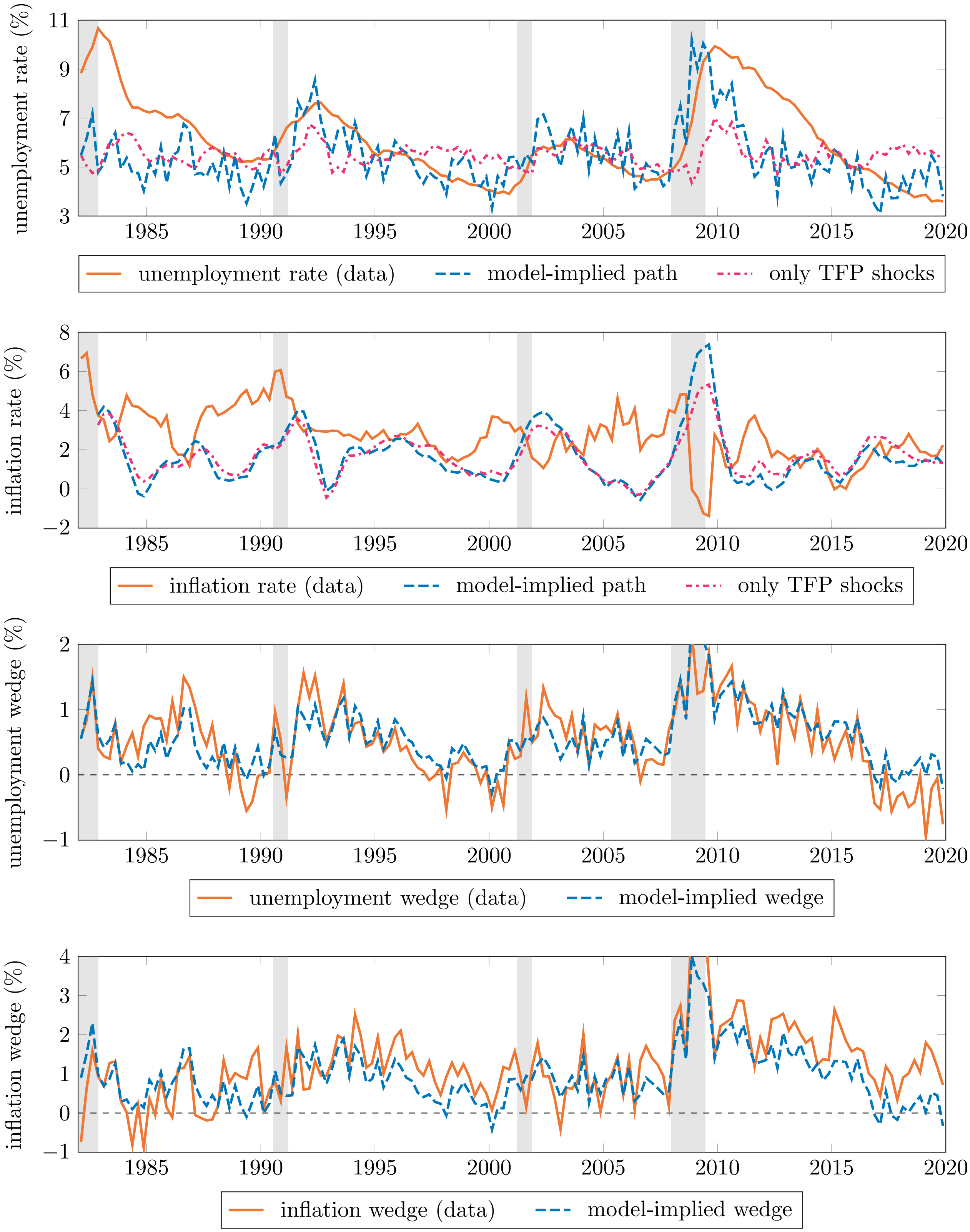


**Figure 6:** Data and model-implied time series for unemployment, inflation, and belief wedges.

exercise in which we set $\theta_t = \mu_\theta$ and hence shut down fluctuations in belief biases.

The top panel shows that adding belief shocks to the model moves the model-implied time series much closer to the data. In particular, without fluctuations in $\theta_t$, the model overstates the unemployment during the dot-com boom in the late 1990s and understates the unemployment around the Great Recession. The model interprets the decrease in unemployment in the late 1990s as arising from relative optimism among households and attributes much of the increase in unemployment around 2008–2009 to an increase in pessimism. The second panel represents the model-implied path for inflation. As expected from the preceding discussion, the model implies countercyclical inflation even though inflation is acyclical in the data. However, fluctuations in $\theta_t$ have a negligible impact on the inflation path. As we elaborate in Section 5, an increase in pessimism acts as a contractionary demand shock while increasing firms' concerns about high future prices, leading to only a small equilibrium impact on inflation.

The bottom two panels of Figure 6 show that the time variation in $\theta_t$ generating the plotted path for unemployment also implies belief wedges that closely match the data. Time variation in $\theta_t$ plays a substantial role in explaining fluctuations in unemployment, but does not act in isolation. For instance, when we shut down TFP and monetary policy shocks (setting $\sigma_a = \sigma_r = 0$), the belief wedges and macro aggregates exhibit no volatility (see the fourth column in Table 2, labeled "only $\theta_t$"). In this equilibrium, households do not face uncertainty about continuation values and do not form distorted expectations about the future path of the economy.[19]

The fifth column (no $a_t$) of Table 2 shows the results in the economy without TFP shocks ($\sigma_a = 0$) and reveals a key interaction between fluctuations in $\theta_t$ and other structural shocks that is required to rationalize the observed belief wedges. To make the economy comparable, we recalibrate the process for $\theta_t$ to fit the properties of the unemployment wedge from the benchmark model (setting $\mu_\theta = 150$ and $\sigma_\theta = 117.7$). The model misses properties of the belief wedges despite generating a sizable amount of volatility in macroeconomic aggregates. The inflation wedge now has a substantially lower volatility, a negative mean, and a negative correlation with the unemployment wedge, all of which are inconsistent with the empirical evidence documented in Section 2.

In the absence of TFP shocks, pessimistic households are concerned about adverse realizations of monetary policy and belief shocks. Both of these shocks act as demand-type shocks, simultaneously lowering economic activity and inflation. Low realizations of households' continuation value are therefore associated with high unemployment and low inflation, moving the two respective belief wedges in opposite directions.

In the sixth column, we present results for the specification of subjective beliefs where $\theta_t$ is driven by fluctuations in TFP, $\theta_t = \theta(a_t)$. We discuss this specification in Section 5.3.

Our specification of the labor market involves Nash bargaining with rigid wages in the spirit of Shimer (2010). The last column of Table 2 presents the results for the case when wage rigidity is removed from the model ($\lambda = 0$) and firms and workers use the standard Nash bargaining

[19] The nonstochastic equilibrium is self-confirming in the sense of Fudenberg and Levine (1993). Households still distort the future distribution of the process $\theta_t$, but that is irrelevant for their decisions.

protocol. Without wage rigidity, both TFP and belief shocks are less potent in their ability to affect macroeconomic aggregates. The magnitude of macroeconomic fluctuations declines, with unemployment volatility falling by almost a half. The decrease in macroeconomic volatility reduces the covariance of forecasted variables with continuation values, and the mean and volatility of the belief wedges decreases as well.[20]

# 5 Understanding the role of subjective beliefs

In this section, we analyze two types of dynamic responses to innovations in $\theta_t$: those under the data-generating measure $P$ and those under the subjective belief $\widetilde{P}$. Together they clarify the mechanism through which fluctuations in subjective beliefs propagate in the economy.

## 5.1 Belief wedges and dynamic responses under subjective beliefs

Figure 7 depicts the impulse responses to an innovation $w_t^\theta$ under the data-generating measure. An increase in $\theta_t$, depicted in the bottom right graph, is contractionary. A one standard deviation innovation to $\theta_t$ leads on impact to a fall of about 1% in output and a 1 percentage point increase in the unemployment rate. Inflation increases on impact for a short period but decreases afterward, with a 10-quarter cumulative response of approximately zero. The contractionary effects of an increase in $\theta_t$ are about two-thirds of the response to a typical productivity shock: a one standard deviation fall in productivity leads to a cumulative 10-quarter decrease of 1.2% in annual output, compared with a 0.8% decrease in the case of a belief shock. The bottom panels of Figure 7 show that households also increase their upward bias in inflation and unemployment forecasts relative to the data-generating measure, which is consistent with the survey data described in Section 2.

The dynamic responses of the exogenous shocks under households' subjective measure $\widetilde{P}$ are shown in Figure 8. Under the data-generating measure $P$ (dashed line), innovations to individual exogenous shock processes are uncorrelated and iid over time, and hence the technology and monetary policy shocks do not respond to $w_t^\theta$. In contrast, under the households' subjective measure $\widetilde{P}$ (solid line), the shocks are correlated. Households associate an increase in $\theta_t$ with a negative productivity shock accompanied by a monetary tightening. They also forecast a further sequence of positive innovations to $\theta_t$, hence increasing the subjective persistence of the belief shock. The particular correlation structure arises through the effect that these three innovations have on the continuation value $V_t$. The pessimistic household also distorts productivity shocks more than monetary policy shocks, reflecting a more adverse impact of the productivity shock on the continuation value.

The continuation value recursion (19) indicates that bad times must be generated by low levels of current and future consumption under the households' subjective model. The top left panel of Figure 9 confirms this intuition. Households facing an increase in $\theta_t$ forecast a large and very persis-

[20]In this exercise, we keep all other parameters of the model unchanged. We further analyze recalibrated versions of the flexible wage model in Appendix F.3.

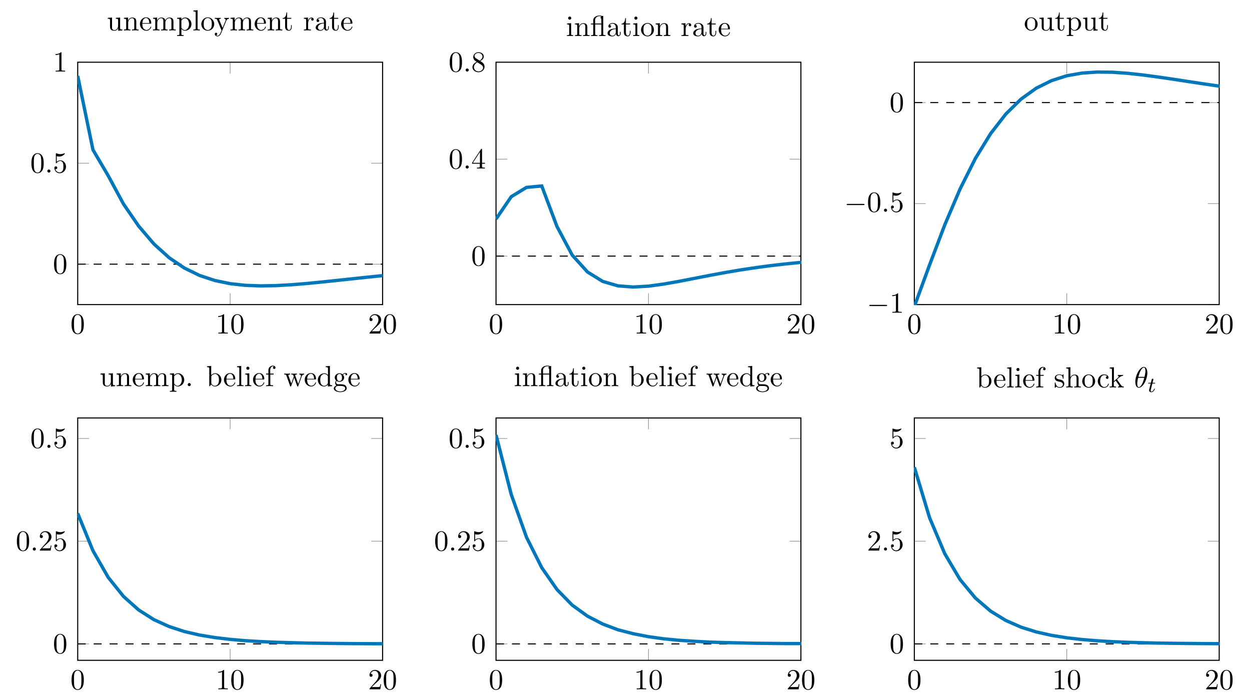


**Figure 7:** Impulse response functions to the belief shock innovation $w^\theta$ under the data-generating measure $P$. Output response is in percentages, and unemployment rate, inflation rate, and belief wedges are in percentage points. Inflation rate is the 4-quarter change in the price index. Horizontal axis is in quarters.

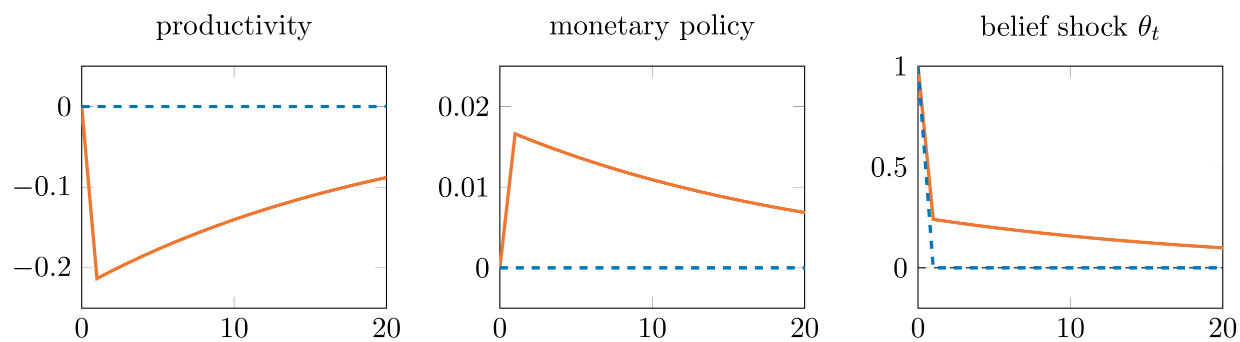


**Figure 8:** Impulse response functions of exogenous shocks to the belief shock innovation $w^\theta$ under the data-generating measure $P$ (dashed line) and the subjective measure $\widetilde{P}$ (solid line). Horizontal axis is in quarters.

tent drop in consumption (solid line) relative to the data-generating process (dashed line). A higher subjective persistence of bad times under the pessimistic belief is manifested in all macroeconomic quantities.[21]

The equilibrium mapping from exogenous shocks to endogenous variables also explains why households forecast higher unemployment, lower output growth, and higher inflation relative to the data-generating process. When TFP shocks are sufficiently prominent, households' inflation

[21] These findings are consistent with Piazzesi et al. (2015), who find higher persistence in the survey forecasts of interest rates in the Blue Chip Financial Forecasts data.

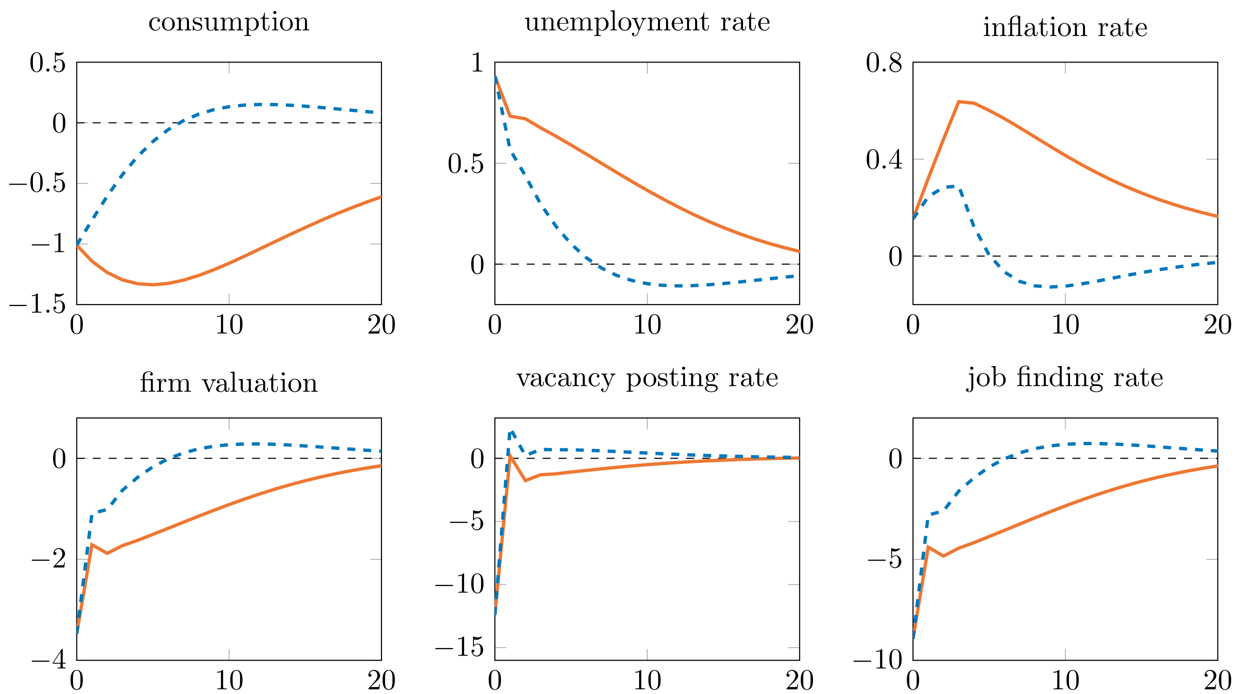


**Figure 9:** Impulse response functions to the belief shock innovation $w^\theta$ under the data-generating measure $P$ (dashed line) and the subjective belief $\widetilde{P}$ (solid line). Responses are in percentages, except for the unemployment rate and inflation rate, which are in percentage points. Inflation rate is the 4-quarter change in the price index. Horizontal axis is in quarters.

expectations increase relative to the rational forecast because expectations of lower productivity imply higher marginal costs, which pushes prices upward through the optimal pricing behavior of firms. The top right panel of Figure 9 shows that households fear persistently higher inflation in the future. As in the Bank of England Inflation Attitudes Survey depicted in Figure 2, they associate adverse states with high inflation.

Figure 9 also shows that the increase in $\theta_t$ has a particularly pronounced contractionary effect on labor market dynamics. Firm valuation $J_t$, given by the present discounted value of profits earned by the firm from a match with a worker,

$$J_t = \vartheta_t - \xi_t + \rho \widetilde{E}_t \left[ s_{t+1} J_{t+1} \right],$$

decreases substantially under the more pessimistic belief in $\widetilde{E}_t [\cdot]$, which causes a large drop in vacancy-posting rates $v_t$ and job-finding rates $f_t$. Again, adverse labor market conditions are expected to last significantly longer under the subjective belief.

The mechanism through which subjective beliefs alter forward-looking decisions in our model provides an interpretation of Euler equation wedges featured in a range of papers as a source of aggregate fluctuations. For example, Smets and Wouters (2007) introduce a “risk premium” shock that has been shown to play an important role in the post-2008 dynamics, Basu and Bundick (2017) and Hall (2017) include direct shocks to the discount rate, and Leduc and Liu (2016) use second-moment shocks to TFP. We share the idea that fluctuations in the consumption Euler equation wedge affect aggregate demand and that wedges in firms' Euler equations affect price setting and match creation. Our framework integrates these mechanisms through movements in subjective beliefs that are disciplined by survey data.

In addition, the importance of uncertainty about supply-type shocks that arises endogenously when we match survey evidence provides a rationale for the specification in Ilut and Schneider (2014). They use a model of exogenously specified time-varying ambiguity aversion about the TFP process based on the multiple-prior preferences of Gilboa and Schmeidler (1989) and Epstein and Schneider (2003). Agents in their model behave *as if* endowed with a subjective belief that exhibits time-varying pessimism about TFP shocks. We show that these types of belief distortions are needed to match survey data jointly with macroeconomic outcomes. More generally, our framework uses endogenous exposures of the continuation value as inputs to belief distortions and thereby avoids overparameterization. This approach is particularly useful in settings with multiple exogenous shocks.

We have shown that one can jointly match macroeconomic and belief wedge data using a relatively parsimonious calibrated model. However, we do not rule out a possible role for additional sources of subjective biases, or the potential presence of other shocks and frictions. In an earlier version of this paper (Bhandari et al. (2016)), we studied an estimated model with a substantially richer environment, reaching the same conclusions about the role of fluctuations in subjective beliefs.

### 5.2 Role of firms' subjective beliefs

The benchmark economy features homogeneous subjective beliefs imposed on all agents and their forward-looking decisions: the consumption-saving decision represented by the consumption Euler equation, the dynamic pricing behavior of intermediate goods producers that determines the New Keynesian Phillips curve, vacancy posting decisions of firms, and bargaining between firms and workers in the labor market, driven by valuation of firms' and workers' surpluses from created matches.

At the same time, there are plausible reasons why beliefs of households and firms may differ. Candia et al. (2020) summarize the available evidence from household and firm surveys for a variety of countries and conclude that while households robustly associate high inflation with adverse outcomes, the evidence for firms is more mixed. In some surveys, firms' forecasts are closer to households' forecasts, while in others, they are closer to those of professional forecasters.[22]

To uncover the role played by the assumption that firms inherit the subjective beliefs of the households, we now study a variant of the model in which we impose rational expectations on firms while preserving subjective beliefs for the households. To implement this variant, we exploit the tractability of our framework to solve for an equilibrium in which we "turn off" belief distortions on specific forward-looking equations. Formally, we look for the solution to the system of equations

[22] Chahrour and Ulbricht (2023) also study fluctuations in beliefs or confidence that are specific to firms and quantify their impact on macroeconomic dynamics.

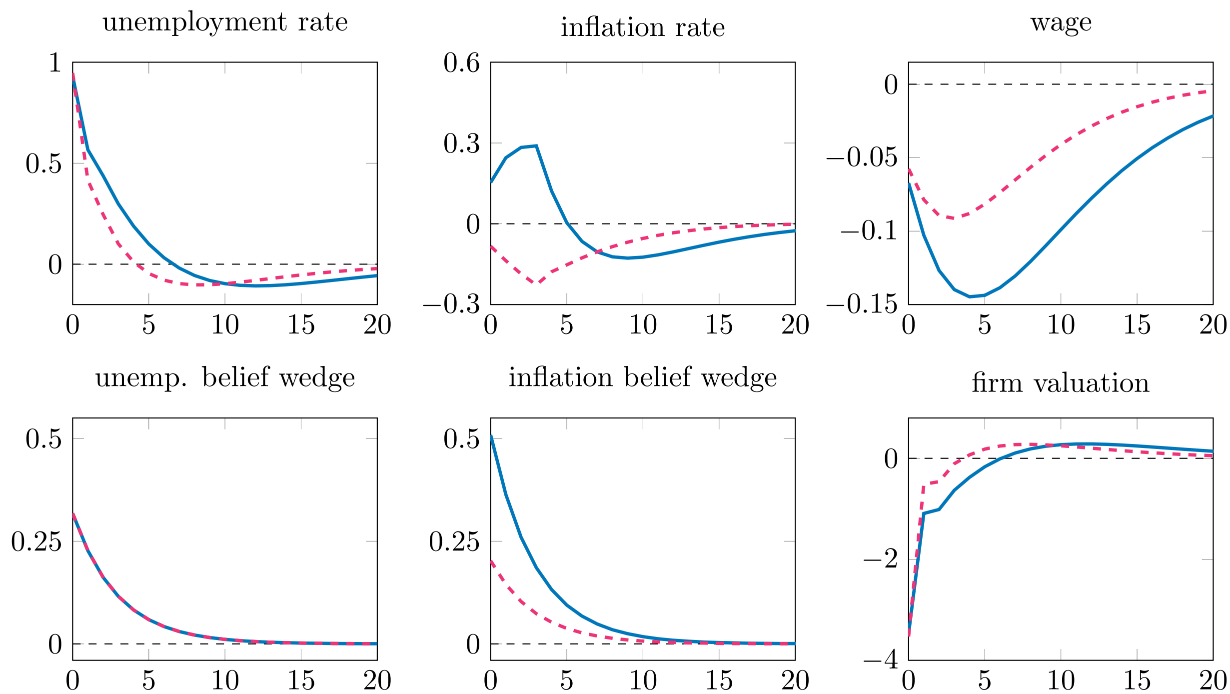


**Figure 10:** Impulse response functions to the belief shock innovation $w^{\theta}$ in the benchmark model with homogeneous subjective beliefs (solid line) and in an economy with rational firms (dashed line). Wage and firm valuation responses are in percentages, and unemployment rate, inflation rate, and belief wedges are in percentage points. Inflation rate is the 4-quarter change in the price index. Horizontal axis is in quarters.

(7) modified as follows:

$$0 = E_t \left[ \mathbb{M}_{t+1} g \left( x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t \right) \right],$$

where $g$ is, as before, the $n \times 1$ vector of functions that includes forward-looking Euler equations and other equilibrium conditions, and $\mathbb{M}_{t+1} \equiv \operatorname{diag}\left\{ m_{t+1}^{\sigma_1}, \ldots, m_{t+1}^{\sigma_n} \right\}$ are the separate belief distortions on each of the $n$ equations. We consider two distinct belief distortions $\sigma_i \in \{0, 1\}$. The expression $m_{t+1}^{0} \equiv 1$ denotes an undistorted equation under rational expectations, and

$$m_{t+1}^{1} \equiv \frac{\exp\left(-\theta_t V_{t+1}\right)}{E_t \left[\exp\left(-\theta_t V_{t+1}\right)\right]}$$

denotes, as in (5), an equation under the subjective belief. For a given configuration of $\{\sigma_i\}$, we solve for new equilibrium dynamics and the associated continuation process $V_{t+1}$. To make the role of subjective beliefs in these economies comparable, we adjust the mean and volatility parameters of the exogenous process $\theta_t$ such that the mean and volatility of the unemployment wedge are unchanged. For details on implementation and a more general treatment of heterogeneous beliefs in this framework, see Appendix B.5.

Figure 10 depicts the dynamic responses in the variant with rational firms (dashed line) and compares them with the benchmark in which all agents have subjective beliefs (solid line). The model with rational firms produces similar fluctuations in unemployment as the benchmark but markedly different dynamics for inflation and wages—inflation is lower on impact, and wages fall by less. Intermediate goods firms with rational beliefs realize that an increase in $\theta_t$ is contractionary but, unlike their counterparts in the benchmark model, do not associate it with higher future marginal costs and therefore do not increase prices as much.[23] Unconditional moments emerging from this calibration are reported in the last column of Table 2. A notable difference is the much smaller volatility of the inflation wedge relative to the baseline calibration.

The reduced fall in wages arises because of the asymmetry in beliefs of firms and workers in the labor market.[24] Under the Nash protocol, firms and workers split the surplus, which is the difference between firms' subjective valuation of output produced in the match and workers' subjective value of unemployment. When firms have rational beliefs, an increase in $\theta_t$ does not alter their valuation of the match output as much as it would have if they shared the pessimistic beliefs of the workers. As a result, the subjective surplus that is bargained over is higher, and the wage that implements the bargaining outcome falls by less.

The drop in inflation in response to an increase in $\theta_t$ implies that inflation comoves less strongly with adverse states. As a result, even though we recalibrated the volatility of $\theta_t$ innovations to match the volatility of the unemployment wedge from the benchmark model, the model with rational firms predicts substantially lower volatility of the inflation wedge. Fluctuations in subjective beliefs on the side of the intermediate goods firms therefore constitute an essential ingredient to reconcile the inflation survey data and macroeconomic outcomes.

### 5.3 Fluctuations in pessimism induced by productivity shocks

We now relax the assumption that time variation in $\theta_t$ is orthogonal to other shocks under the data generating process. We explore the implications of two extensions in which belief fluctuations are induced by other structural shocks in the model and by time-variation in risk exposures that the homoskedastic model abstracts from.

In particular, we first study a version of the model in which fluctuations in $\theta_t$ are induced by innovations to TFP $a_t$:

$$\theta_t = \mu_\theta + c_a a_t. \tag{21}$$

In contrast to the benchmark with $\theta_t$ that follows an AR(1) process with independent innovations, subjective biases are affected directly by productivity shocks and feed back through the decisions of forward-looking consumers, firms, and workers, amplifying the overall effect of the initial impulse to productivity. This captures a common narrative of extrapolative expectations (see Adam et al. (2021), Gennaioli et al. (2015), Nagel and Xu (2022), and references therein). With $c_a < 0$, a negative innovation to TFP results in higher pessimism, and agents expect unemployment and inflation to be high relative to what is implied by the data generating process. This leads to higher

[23] The distorted beliefs of the price-setting firms following a large increase in $\theta_t$ (see Figure 6) can thus also account for the "missing disinflation" during the Great Recession, discussed in Coibion and Gorodnichenko (2015b).

[24] In the bargaining process, firms and workers agree to disagree about their subjective valuation of the match, in the sense of Harrison and Kreps (1978) and Morris (1995).

future unemployment compared to the model with rational expectations. Conversely, a positive innovation to TFP propagates through optimism about the future.

The last column ($\theta(a_t)$) in Table 2 shows that the model with fluctuations in $\theta_t$ induced by TFP is able to match the moments for both macroeconomic aggregates and the belief wedges about as well as the baseline exogenous $\theta_t$ model. In particular, the amplification from households' subjective beliefs allows the model to match the volatility of unemployment in the data using only TFP and monetary policy shocks. However, these unconditional moments are now generated by conditional responses of unemployment to TFP shocks that are substantially larger than suggested by the VAR evidence discussed in Section 4.2. Figure 26 in Appendix F.2 shows the impulse response to a negative TFP shock and compares it to the TFP response in an economy without the feedback through subjective beliefs ($c_a = 0$).

Moreover, the fit of the simulated paths for unemployment and the belief wedges, shown in Figure 27 in Appendix F.2, worsens significantly. In particular, the correlation between the data and the model-implied paths for unemployment, the unemployment belief wedge, and the inflation belief wedge are 0.18, 0.25, and 0.45 in the model with $\theta_t$ induced by TFP, as compared to 0.48, 0.79, and 0.76 in the benchmark model.

We view the above differences in model fit as evidence for quantitatively important movements in $\theta_t$ that are orthogonal to productivity. Richer specifications of how fluctuations in pessimism are shaped by the evolution of other shocks as well as endogenous variables is left for future research.

### 5.4 Idiosyncratic risk as a source of endogenous fluctuations

The linear model approximation we construct abstracts from time-variation in risk induced by heteroskedastic innovations or model non-linearities. Specifically, the continuation value approximation (14) is a linear function of the process $x_{1t}$ that follows a homoskedastic vector-autoregression. Consequently, the only source of variation in belief wedges (18) are movements in $\theta_t \approx \overline{\theta} x_{1t}$.

While the model in Section 5.3 featured time variation in $\theta_t$ driven by TFP shocks, we now consider a model where $\theta_t$ is constant but belief fluctuations are induced by time-varying uninsurable idiosyncratic risk.[25] We follow the framework of Constantinides and Duffie (1996) and postulate an endowment economy populated by a unit mass of households indexed by $i \in [0, 1]$, with consumption of household $i$ given by $C_t^i = \delta_t^i C_t$. The share processes $\delta_t^i$ are modeled as

$$\frac{\delta_{t+1}^i}{\delta_t^i} = \exp\left(-\eta_{t+1}^i \sigma_{t+1} - \frac{1}{2}\sigma_{t+1}^2\right),$$

where $\eta_{t+1}^i \sim N(0, I)$ are permanent growth shocks to individual consumption that are independent in the cross section and of aggregate variables. The variance process $\sigma_t^2$ that controls the dispersion

[25] Mitra and Xu (2023) study a search model with firms that are ambigious about an idiosyncratic match-specific component of productivity.

of idiosyncratic uncertainty follows

$$\sigma_{t+1}^2 = (1-\psi_\sigma)\,\bar{\sigma}^2 + \psi_\sigma \sigma_t^2 + \sigma_t \psi_{\sigma w} w_{t+1},$$

with $\psi_{\sigma w}$ being a $1 \times k$ vector of exposures to aggregate macroeconomic shocks specified in (4).[26] Household's period utility function is logarithmic, $u(C) = (1-\beta)\log C$. Households are allowed to trade claims that contingent on aggregate states but there are no markets that can insure idiosyncratic risk. To keep the framework analytically tractable, we abstract from variation in aggregate consumption and assume $\log C_{t+1} - \log C_t = \bar{c}$.

In Appendix G, we show that under these assumptions, there is no inter-household trade in claims contingent on aggregate risk, and households share a common subjective belief with respect to aggregate variables given by the belief distortion

$$\widehat{m}_{t+1} = \frac{\exp\left(-\theta\left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right)\sigma_t \psi_{\sigma w} w_{t+1}\right)}{E_t\left[\exp\left(-\theta\left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right)\sigma_t \psi_{\sigma w} w_{t+1}\right)\right]},$$

where $\bar{v}_\sigma$ is the exposure of the household's continuation value to the variance process $\sigma_t^2$.

Consider an aggregate variable $z_t = \bar{z}' x_{1t}$ where $x_{1t}$ follows the linear process (10). Then the belief wedge, as the difference between the subjective forecast and forecast under the data-generating measure, is given by

$$\begin{aligned}\Delta_t^{(1)}(z) &= \widetilde{E}_t[z_{t+1}] - E_t[z_{t+1}] = \bar{z}'\psi_w \widetilde{E}_t[w_{t+1}] \\ &= -\theta\sigma_t \bar{z}'\psi_w \psi_{\sigma w}' \left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right).\end{aligned}$$

This is a direct counterpart of expression (18) from the representative household model. While the dynamics of the aggregate variables are homoskedastic and the penalty parameter $\theta$ is constant, time-variation in idiosyncratic risk induces time-varying belief distortions. Since the term in the parenthesis is negative, times of high idiosyncratic uncertainty are bad for the household. Consequently, households overpredict variables that comove positively with innovations to dispersion in idiosyncratic uncertainty, as indicated by the covariance term $\bar{z}'\psi_w\psi_{\sigma w}'$.

In Figure 11, we use the idiosyncratic skewness index constructed by Schmidt (2016) as a measure of dispersion of idiosyncratic risk. The skewness index emphasizes the time-varying nature of the likelihood of large adverse idiosyncratic shocks like job loss, which are empirically only imperfectly insured by households or labor market institutions. The blue dashed line represents the negative of the standardized index, while the solid orange line is the first principal component of the belief wedges.

Households are more pessimistic during times of high idiosyncratic risk, and the correlation

[26] With Gaussian shocks $w_{t+1}$, the linear variance process can take negative values, which can be avoided either with an appropriate truncation of the shocks, or by considering a continuous-time limit, which yields a strictly positive square root process used in Cox et al. (1985). The variance process could be extended to include exposure to a persistent vector-autoregression.

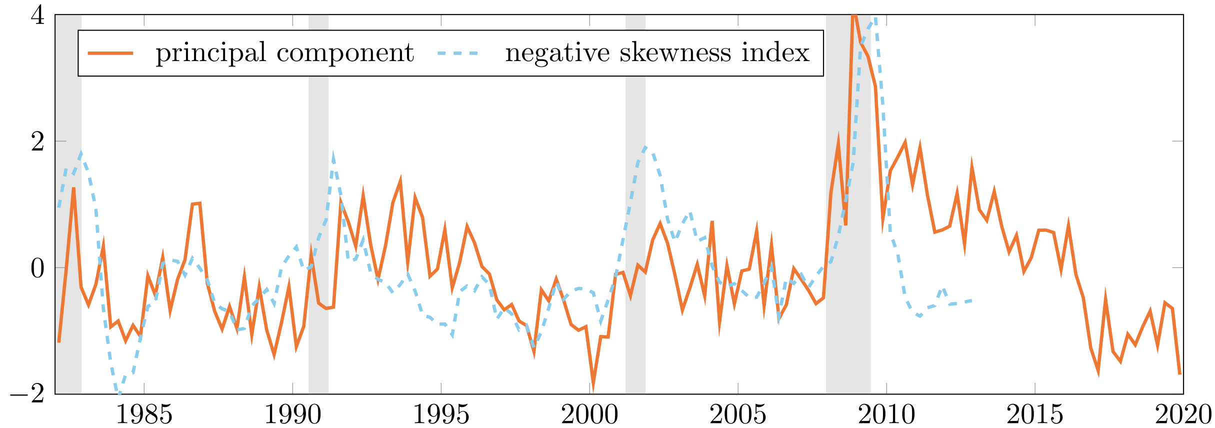


**Figure 11:** Comparison of the first principal component of the belief wedges (orange solid line) with the index of idiosyncratic skewness constructed by Schmidt (2016) from cross-sectional Social Security Administration data on labor income growth rates (dashed blue line). Both time series are standardized and the skewness measure is plotted with a negative sign. NBER recessions are shaded.

between the two time series is $-0.49$. Providing a quantitative link between the skewness index and the belief wedges would require a complete description of the economy which is beyond the scope of this paper but the strong comovement supports the hypothesis that fluctuations in idiosyncratic uncertainty can be a meaningful rationale for the time variation in pessimism.

# 6 Verification of the mechanism

In this section, we provide further evidence that compares predictions of the structural model with the data and verifies the quantitative importance of belief fluctuations for macroeconomic dynamics. First, we use local projections to construct empirical impulse responses of macroeconomic variables and survey forecasts to innovations in the belief wedge $\theta_t$ and compare them to their model counterparts. Second, we show that the structural model produces forecast errors in subjective beliefs that align with those in the household survey data. Finally, we show that the time series for $\theta_t$ extracted from belief wedges is highly correlated with empirical proxies for consumer confidence and discuss advantages of using survey forecasts data over the confidence proxies.

## 6.1 Impulse responses constructed using local projections

We first provide additional evidence for our mechanism by comparing the model-implied impulse responses to the belief shock $\theta_t$ with corresponding impulse responses estimated using the flexible local projection approach proposed in Jordà (2005). Our structural model imposes an AR(1) structure on the belief shock $\theta_t$, which is connected to the endogenous variables through the cross-equation restrictions, with linear Markov dynamics obtained as the solution of the general equilibrium model. The local projections relax the cross-equation restrictions and the linear Markov dynamics from our model, thus providing further reduced form evidence supporting our model's implications about

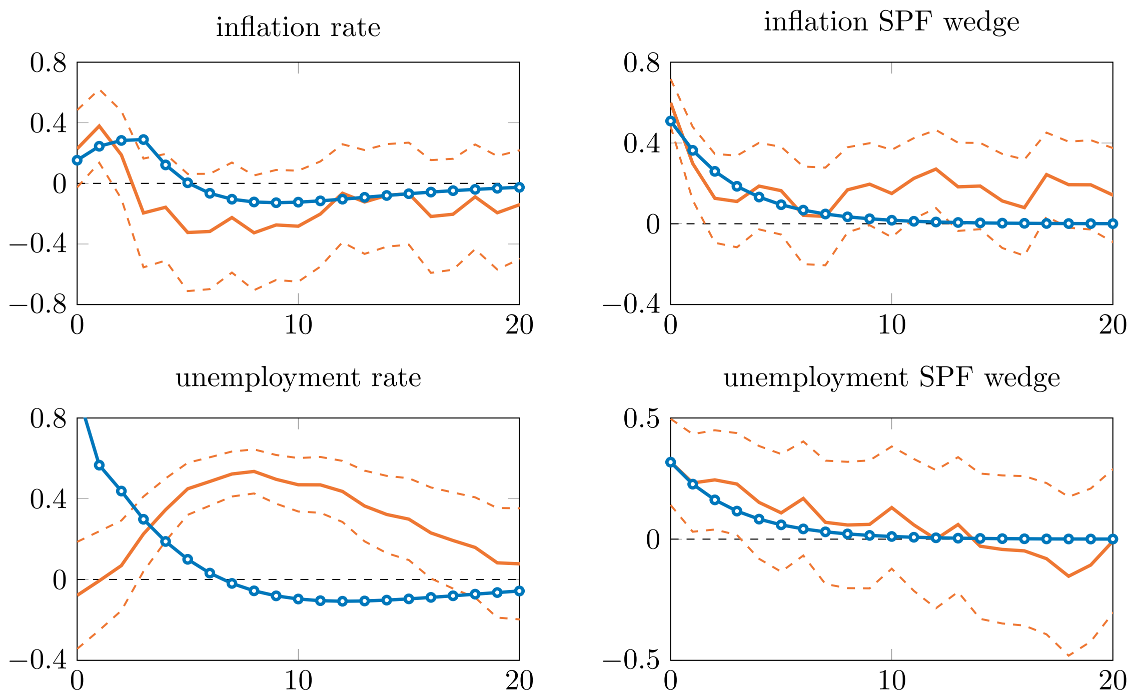


**Figure 12:** Impulse response functions of Michigan Survey responses to the innovation in the principal component of the belief wedges (solid orange lines), constructed as the difference between the Michigan and SPF forecasts. Responses are in percentage points, inflation rate is annualized. Dashed lines represent $\pm 1$ standard deviation bands constructed using heteroskedasticity-robust standard errors. Model-implied impulse responses are displayed using blue lines with circles. Horizontal axis is in quarters.

the effects of fluctuations in pessimism and optimism.

We follow the lag augmentation procedure advocated by Montiel Olea and Plagborg-Møller (2021), and estimate the impulse response to a variable of interest $z_t$ as the function $\beta_z(h)$, $h = 0, \ldots, H$ using the regression

$$z_{t+h} = \beta_z(h)\,\theta_t + \sum_{l=1}^{p} \gamma_{z,l}(h)\,\theta_{t-l} + \xi_t(h)\,.$$

Under our model assumptions, these impulse responses correspond to the impulse responses to the shock $w_t^\theta$ analyzed in Section 5. Montiel Olea and Plagborg-Møller (2021) show validity of Huber–White heteroskedasticity robust standard errors for this regression under the assumption that $\theta_t$ follows an autoregressive process up to order $p$.

In order to avoid any potential concerns related to the misspecification of the belief wedges due to the estimation of the VAR forecasts, we estimate in this exercise impulse responses to innovations to the belief shock $\theta_t$ that is constructed as the principal component of the belief wedges between the Michigan and SPF forecasts. In Appendix F.1, we show that the principal component is very close to the filtered path of $\theta_t$ estimated in a hidden state space model. Results for the belief shock $\theta_t$ constructed from the belief wedges between the Michigan and VAR forecasts are analogous.

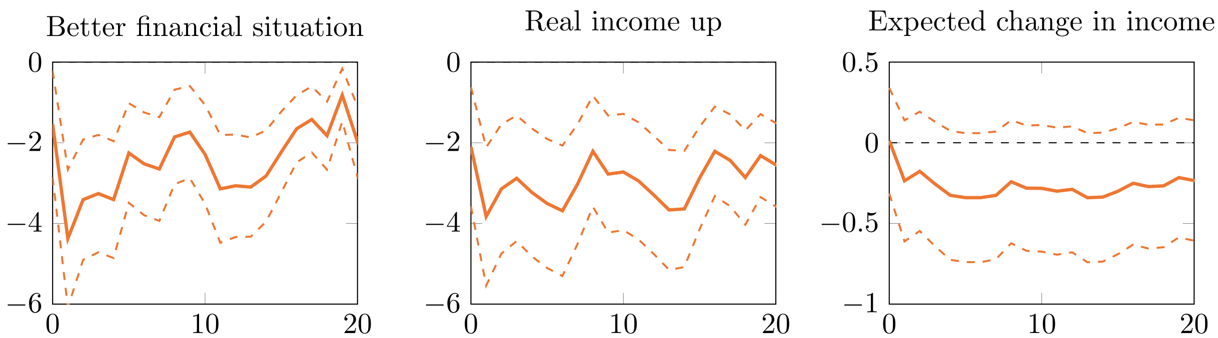


**Figure 13:** Impulse response functions of Michigan Survey responses to the innovation in the principal component of the belief wedges, constructed as the difference between the Michigan and SPF forecasts. *Better financial situation*: Net share of households expecting better financial conditions in one year (percentage points). *Real income up*: Net share of households expecting higher real income in one year (percentage points). *Expected change in income*: Expected percent change in income in one year for the median household. Dashed lines represent $\pm 1$ standard deviation bands constructed using heteroskedasticity-robust standard errors. Horizontal axis is in quarters.

Figure 12 depicts the responses of the belief wedges, and actual inflation and unemployment for the local projections and the model.[27] Overall, the two approaches produce similar responses. In response to a positive innovation to the belief shock (i.e., an increase in $\theta_t$), both the local projections and the structural model find a substantial increase in the unemployment rate and an initial brief increase followed by a modest but persistent decrease in the inflation rate. While the structural model does not have a mechanism to generate the hump-shaped response of unemployment estimated by the local projections, the overall magnitudes of the impulse responses are comparable. These responses correspond to an increase in pessimism, with both the unemployment and inflation wedges increasing. In Appendix E.2, we also construct the predicted responses of the inflation and unemployment forecasts, and for wedges constructed using VAR forecasts as robustness checks.

Figure 13 shows that an increase in the belief shock $\theta_t$ also predicts a pessimistic shift in households forecasts of other quantities. The net shares of households expecting better financial conditions and higher real income in one year decrease. Expected growth in nominal income for the median household decreases as well. These aggregate responses corroborate the micro-level evidence from the panel data in the Michigan and FRBNY Surveys that we discussed in Section 2.2 and detailed in Appendix D.5.

## 6.2 Forecast error regressions

As another test of our theoretical mechanism, we now show that the dynamics of subjective beliefs implied by our model are also consistent with the dynamics of forecast errors measured in the survey data. To that end, we estimate a linear model of forecast errors for a variable of interest $z_t$

[27] It is well known that confidence intervals for the impulse responses constructed using local projections can be wide (see for example Li et al. (2021) and Miranda-Agrippino and Ricco (2021)), and literature has proposed shrinkage procedures for standard errors. We follow a conservative approach here, without implementing any shrinkage. Nevertheless, the estimated responses are instructive for verifying the mechanism in our model.

using regressions of the form

$$z_{t+j} - \widetilde{E}_t [z_{t+j}] = b_0 + b_z z_t + b_f \widetilde{E}_{t-1} [z_{t+j-1}] + \varepsilon_{t+j}, \tag{22}$$

where $z_t \in \{\pi_t, u_t\}$, and $\widetilde{E}_t [z_{t+j}]$ is the subjective forecast at time $t$ for the realization $j$ periods ahead. We formally specify assumptions to derive this updating rule in Appendix H. Under full information rational expectations, we should expect the errors to be mean zero and unforecastable, $b_0 = b_z = b_f = 0$.

Our model does not imply equation (22) as a structural relationship for belief updating, since the state space that determines the relationship between forecast errors and predictors involves multiple exogenous and endogenous variables. However, given the prevalence of such univariate predictive regressions in empirical work, it is still informative to run the reduced-form regressions on observed data and compare them with regressions using model-simulated data as a further validation of our mechanism.[28]

Table 3 compares our empirical estimates to the corresponding values obtained from data simulated from our model. The top panel reports results for inflation forecasts, while the bottom panel for unemployment forecasts. We estimate the multivariate regression (22) as well as two univariate regressions with each of the regressors separately. The regressors are demeaned, hence the estimated coefficient $b_0$ corresponds to the mean of the forecast error on the left-hand side.

First, as documented in Section 2, the data indicate that on average, households overpredict inflation and unemployment by a significant amount, hence the negative coefficients $b_0$ in the first row of the table. As we have already seen, our model is consistent with these large average biases. Our model also generates the right amount of predictability in inflation and unemployment forecast errors, reflected by very similar $R^2$ coefficients in the empirical and theoretical regressions.

Importantly, our model delivers a distinct predictability pattern for inflation and unemployment forecasts, in both cases consistent with the data. The significant negative coefficient $b_f$ in the inflation regression indicates that inflation forecast errors are persistent—high previous inflation forecasts predict excessively high forecasts going forward. On the other hand, predictability of unemployment rate forecast errors is dominated by the current state—when unemployment is high, households are particularly pessimistic and predict overly high unemployment going forward.

Introduction of the belief shock to a standard DSGE model thus delivers both the mean biases and predictability patterns of households' inflation and unemployment forecasts as a joint outcome. Rather than updating beliefs about inflation and unemployment independently, the household understands the equilibrium relationship between these two variables but pessimistically biases

[28] Aruoba et al. (2017) advocate comparing estimates of simple univariate time series models using actual and model-generated data as a diagnostic tool for DSGE models. While they focus on assessing nonlinearities generated by model dynamics, we share their aim of using interpretable reduced-form relationships to validate the model. Existing literature (e.g., Coibion and Gorodnichenko (2012, 2015a), Bordalo et al. (2020), Angeletos et al. (2020)) that studies predictability of forecast errors of professional forecasters also often includes forecast revisions $\widetilde{E}_t [z_{t+j}] - \widetilde{E}_{t-1} [z_{t+j}]$ as a regressor. The construction of forecast revisions requires forecasts for multiple horizons ($j$ and $j+1$), which are not available in the Michigan Survey.

| Panel A | (1) | | (2) | | (3) | |
|---|---|---|---|---|---|---|
| Inflation | data | model | data | model | data | model |
| $b_0$ | $-1.26$ | $-0.90$ | $-1.26$ | $-0.90$ | $-1.26$ | $-0.90$ |
| | (0.11) | | (0.11) | | (0.11) | |
| $b_z$ | 0.08 | 0.10 | $-0.12$ | $-0.22$ | | |
| | (0.11) | | (0.08) | | | |
| $b_f$ | $-0.50$ | $-0.47$ | | | $-0.42$ | $-0.38$ |
| | (0.17) | | | | (0.13) | |
| $R^2$ | 0.067 | 0.103 | 0.014 | 0.046 | 0.063 | 0.100 |

| Panel B | (1) | | (2) | | (3) | |
|---|---|---|---|---|---|---|
| Unemployment | data | model | data | model | data | model |
| $b_0$ | $-0.44$ | $-0.55$ | $-0.44$ | $-0.55$ | $-0.44$ | $-0.55$ |
| | (0.10) | | (0.10) | | (0.11) | |
| $b_z$ | $-0.40$ | $-0.46$ | $-0.35$ | $-0.51$ | | |
| | (0.28) | | (0.06) | | | |
| $b_f$ | 0.05 | $-0.10$ | | | $-0.32$ | $-0.52$ |
| | (0.27) | | | | (0.06) | |
| $R^2$ | 0.180 | 0.218 | 0.180 | 0.216 | 0.169 | 0.120 |

**Table 3:** Estimated coefficients for forecast error predictability regression (22) in the data and its theoretical counterpart in the model. Standard errors for the empirical regression are in parentheses. Columns (2) and (3) show results for restricted univariate versions of equation (22) with one of the regressors omitted from the regression.

their joint distribution.

Our results can be contrasted with two prominent frameworks: a Bayesian model with frictions in information processing and a non-Bayesian model of overreaction. The former case is studied by Coibion and Gorodnichenko (2012), who show that various types of information sluggishness, including the sticky information model studied in Mankiw and Reis (2002) or Carroll (2003) or the noisy information model in the spirit of Lucas (1972), Sims (2003), and Woodford (2003b), imply $b_0 = 0$, $b_z \in (0, 1)$, and $b_f \in (-1, 0)$. Coibion and Gorodnichenko (2012) also document this sluggishness for inflation forecasts of households in the Michigan Survey, firms in the Livingston survey, and FOMC members in the Monetary Policy Reports. The latter case is discussed in Kohlhas and Walther (2021) and Angeletos et al. (2020), who argue that overreaction is associated with $b_z < 0$. In particular, new information in $z_t$ implies excessive updating of the forecast $\widetilde{E}_t\left[z_{t+j}\right]$ and hence a negatively correlated forecast error.[29] Both these models also imply that $b_0 = 0$.

[29] Bordalo et al. (2020) provide evidence of such overshooting in the *individual* forecasts in the SPF based on predictability of individual forecast errors using individual forecast revisions, although the *consensus* forecasts tend to respond sluggishly. They rationalize this finding using a model in which forecasters filter a signal contaminated with common and private noise, with a calibration where individual forecast errors are dominated by overreaction to the private noise component, while the average or consensus forecast is driven to sluggish adjustment to the common

The empirical and model-implied predictability patterns summarized in Table 3 cannot be jointly rationalized with either of these models. First, we find $b_0 < 0$ for both inflation and unemployment. In contrast, both models predict that forecasts should be unbiased on average. Second, the slope coefficients in the data row indicate that the evolution of inflation and unemployment forecasts appear to be consistent with different belief updating mechanisms.

On one hand, the inflation regression in column (1) of Panel A is dominated by the large and highly significant negative coefficient on the lagged forecast ($b_f = -0.50$). This suggests that households update inflation forecasts sluggishly, consistent with sticky information models and empirical findings in Coibion and Gorodnichenko (2012). Current inflation does not play a significant role in predicting forecast errors, even in the univariate regression in column (2).

On the other hand, the slope coefficients in the unemployment rate regression in column (1) of Panel B exhibit a distinctly different pattern. The negative coefficient on the current unemployment level ($b_z = -0.40$) suggests that households overreact to changes in unemployment. The magnitude and statistical significance of the overreaction in the univariate regression in column (2) are substantially larger than estimates found by Angeletos et al. (2020) in SPF forecasts using the same relationship.[30]

These results emphasize the importance of departing from univariate models of belief formation toward those that model the joint subjective distribution over the evolution of macroeconomic variables. In our model, belief wedges are cyclical. Since the unemployment rate is a much better predictor of the business cycle than inflation, it is also more successful in predicting forecast errors, aligning empirical evidence with model-implied predictability patterns.

### 6.3 Measures of sentiment and confidence

We have interpreted movements in the belief wedges as arising from time-varying pessimism and optimism. We now show that these are connected more generally with empirical proxies for consumer sentiment, which existing work has also connected to macroeconomic fluctuations.[31] Since these sentiment measures are procyclical, they may be capturing similar demand-driven forces as our mechanism. To understand how these alternative measures relate to our framework, we compare them in Figure 14 with our belief shock $\theta_t$.

The top panel of Figure 14 shows a close connection between the belief shock and the Michigan Consumer Sentiment Index and the Conference Board Consumer Confidence Index. Both indices provide independent measures of sentiment that comove strongly with the belief shock, with correlations of $-0.65$ and $-0.72$, respectively. The comovement is consistent with our interpretation

noise component.

[30]Column (3) shows that the lagged forecast is a statistically significant predictor of the forecast error in the univariate regression. Due to its high persistence, the unemployment rate and its forecast are highly correlated, which makes the forecast insignificant in the multivariate regression.

[31]For example, Barsky and Sims (2012) and Angeletos et al. (2018) use the Michigan Consumer Sentiment Index, and Leduc and Liu (2016) use the share of Michigan Survey households who report that it is not a good time to buy new cars because of an uncertain future. Angeletos et al. (2018) remark on issues with the qualitative nature of the Michigan Consumer Sentiment Index, which our approach addresses.

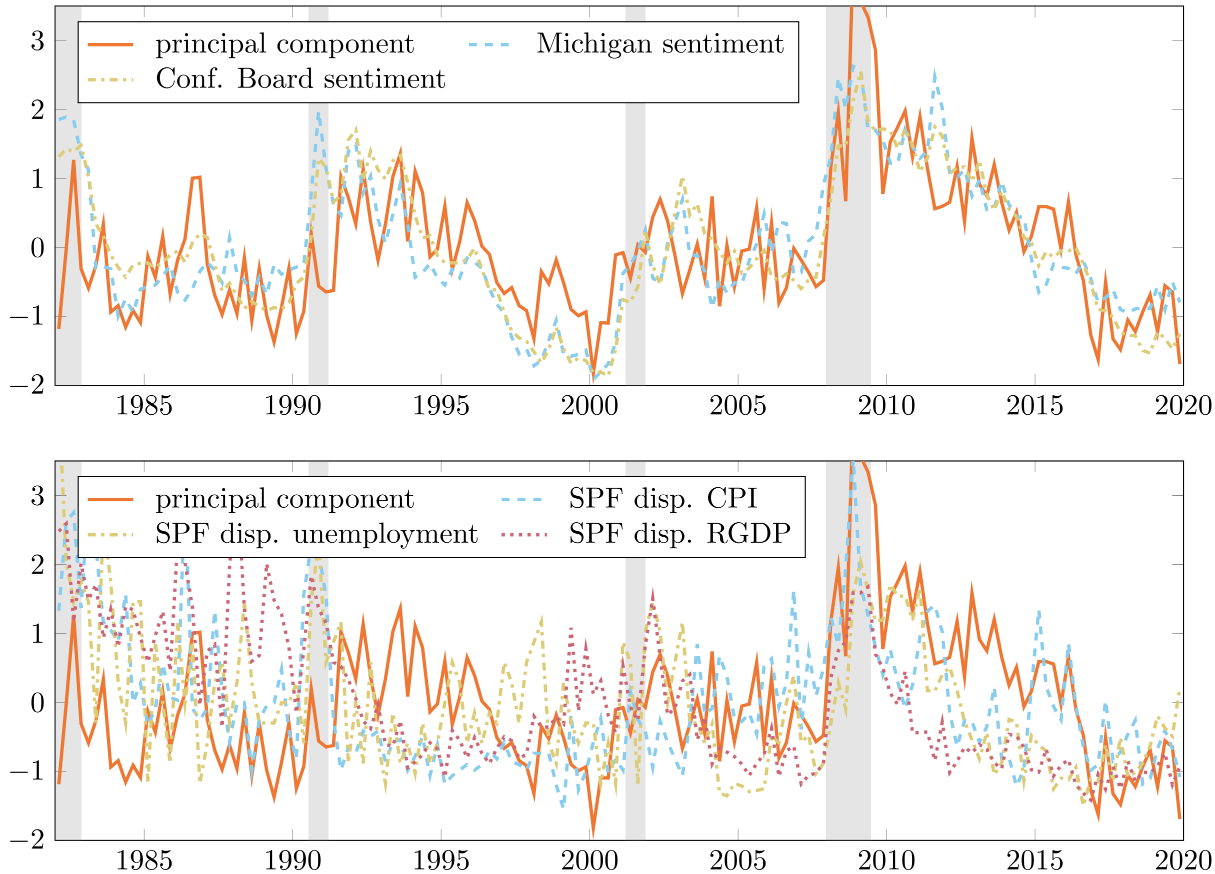


**Figure 14:** Comparison of the first principal component of the belief wedges with alternative measures of sentiment and disagreement. *Top panel:* negative of the Michigan Consumer Sentiment Index, negative of the Conference Board Consumer Confidence Index. *Bottom panel:* Interquartile dispersion in individual forecasts in Survey of Professional Forecasters of CPI inflation, unemployment, and real GDP growth. NBER recessions are shaded. Following Zhao (2017), we average dispersion in SPF forecasts over horizons from zero (nowcast) to three quarters to reduce noise.

of the belief wedges as arising from pessimism, as captured by negative movements in sentiment.

A shortcoming of using empirical measures of sentiment is the lack of theoretical counterparts in quantitative models. The magnitude of fluctuations in these sentiments is thus typically calibrated indirectly using the volatility of macroeconomic quantities. In contrast, we calibrate $\theta_t$ to match the level and volatility of the belief wedges, which we directly measure in survey data, in line with arguments by Dominitz and Manski (2004), Coibion et al. (2018a), and Manski (2017) to use responses to quantitative survey questions in order to provide a tighter link between survey data and theoretical models. Our theory also enforces additional restrictions on the relative magnitudes of the belief wedges, providing overidentifying restrictions that we verify empirically.

Instead of belief wedges, Ilut and Schneider (2014) use the dispersion of SPF forecasts of real GDP as a proxy for the degree of ambiguity on the side of households. In related work, Mankiw et al. (2003), Bachmann et al. (2013), and others use measures of cross-sectional forecast dispersion as a proxy for economic uncertainty, based on the assumption that higher dispersion is indicative of more difficulty in estimating the forecast distribution. The bottom panel of Figure 14 shows that measures of dispersion in SPF forecasts of inflation, unemployment, and real GDP growth are largely uncorrelated with the belief wedges, indicating that disagreement among SPF forecasters is distinct from pessimistic concerns of households. Although our framework and solution method allow for heterogeneity in beliefs that could explicitly model such disagreement in forecasts, we leave a quantitative analysis of such a model for future work.

# 7 Conclusion

In this paper, we develop a framework in which agents' subjective beliefs depart from rational expectations and feature time-varying pessimism and optimism. Using survey data to discipline this departure, we show that subjective beliefs have an economically significant role in driving macroeconomic outcomes, especially labor market quantities.

Pessimistic agents in the model overweight outcomes that are associated with low continuation utilities. Systematic policy changes that alter the distribution of consumption will also affect the distribution of adverse states, and hence agents' subjective beliefs and decisions. We view the policy-invariant nature of the mapping between continuation utilities and beliefs as an extension of the rational expectations hypothesis that preserves immunity to the Lucas critique and makes our framework suitable for the study of normative questions.

A natural application is the conduct of monetary policy where managing private sector expectations stands at the forefront. Another direction is to exploit the cross-sectional differences in beliefs that we documented. With incomplete markets, heterogeneous exposures of continuation values to shocks will generate endogenous heterogeneity in beliefs and has implications for savings, portfolio choices, and labor market behavior. Such a modification of the framework can be applied to study the design of social insurance policies.

# Appendix

## A Subjective beliefs and belief wedges

In this section, we derive formulas for the belief distortions in the linearized version of the dynamic model described in Section 3, extended to include nonstationary shocks as in Appendix B.7. Let $(\Omega, \{\mathcal{F}_t\}_{t=0}^{\infty}, P)$ be the filtered probability space generated by the innovations $\{w_{t+1}\}_{t=0}^{\infty}$, with $w_{t+1} \sim N(0_k, I_{k\times k})$ iid. The subjective probability measure $\widetilde{P}$ is formally defined by specifying a strictly positive martingale $M_{t+1}$ with one-period increments:

$$m_{t+1} = \frac{M_{t+1}}{M_t} = \exp\left(-\frac{1}{2}|\nu_t|^2 + \nu_t' w_{t+1}\right). \tag{23}$$

The conditional mean of the innovation vector under $\widetilde{P}$ then satisfies $\widetilde{E}_t[w_{t+1}] = \nu_t$. We consider linear model dynamics given by

$$\begin{aligned} x_t &= \widehat{x}_t + z_t \\ \widehat{x}_{t+1} &= \psi_q + \psi_x \widehat{x}_t + \psi_w w_{t+1} \\ z_{t+1} - z_t &= \phi_q + \phi_x \widehat{x}_t + \phi_q w_{t+1}. \end{aligned} \tag{24}$$

The vector $x_t$ of economic variables therefore has a stationary component $\widehat{x}_t$ and a nonstationary component $z_t$ that has a stationary growth rate.[32] We impose a restriction on the belief distortion (23):

$$\nu_t = \overline{H} + HF\widehat{x}_t,$$

where $F$ is a $1\times n$ vector and $H$, $\overline{H}$ are $k\times 1$ vectors. The belief distortion derived in the structural model is a special case of this restriction. In particular, in the case of the linear approximation of the stationary model developed in Section 3, we have $z_t \equiv 0$ and $x_{1t} = \widehat{x}_t$. Equation (16) implies that $\nu_t = -\overline{\theta}(\bar{x} + x_{1t})(V_x\psi_w)'$, and hence

$$\overline{H} = -\overline{\theta}\bar{x}(V_x\psi_w)' \qquad H = -(V_x\psi_w)' \qquad F = \overline{\theta}.$$

In the case of the nonstationary model from Appendix B.7, the expressions for $\overline{H}$, $H$, and $F$ are given in equation (58).

Let $\zeta_t = Zx_t = Z(\widehat{x}_t + z_t)$ be an $m\times 1$ vector of variables for which we have observable data on households' expectations where $Z$ is an $m\times n$ selection matrix. We are interested in $\tau$-period-ahead belief wedges

$$\Delta_t^{(\tau)} = \widetilde{E}_t[\zeta_{t+\tau}] - E_t[\zeta_{t+\tau}].$$

Guess that

$$\begin{aligned} E_t[\zeta_{t+\tau} - \zeta_t] &= G_x^{(\tau)}\widehat{x}_t + G_0^{(\tau)} \\ \widetilde{E}_t[\zeta_{t+\tau} - \zeta_t] &= \widetilde{G}_x^{(\tau)}\widehat{x}_t + \widetilde{G}_0^{(\tau)}, \end{aligned}$$

where $G_x^{(\tau)}$, $G_0^{(\tau)}$, $\widetilde{G}_x^{(\tau)}$, and $\widetilde{G}_0^{(\tau)}$ are conformable matrix coefficients with initial conditions

$$G_0^{(\tau)} = \widetilde{G}_0^{(\tau)} = 0_{m\times 1} \qquad G_x^{(\tau)} = \widetilde{G}_x^{(\tau)} = 0_{m\times n}.$$

[32] The linear approximation of the model specified in Section 3 maps directly into this framework. We drop the subindices denoting first-order derivative processes for convenience.

We can then establish a recursive formula for the expectations under the data-generating measure

$$\begin{aligned}
G_x^{(\tau)}\widehat{x}_t + G_0^{(\tau)} &= E_t\left[\zeta_{t+\tau} - \zeta_t\right] = \\
&= E_t\left[Z\left(x_{t+1} - x_t\right) + G_x^{(\tau-1)}\widehat{x}_{t+1} + G_0^{(\tau-1)}\right] \\
&= G_0^{(\tau-1)} + Z\phi_q + \left(Z + G_x^{(\tau-1)}\right)\psi_q + \left[\left(Z + G_x^{(\tau-1)}\right)\psi_x + \left(Z\phi_x - Z\right)\right]\widehat{x}_t \\
&\quad + \left(\left(Z + G_x^{(\tau-1)}\right)\psi_w + Z\phi_w\right)E_t\left[w_{t+1}\right].
\end{aligned} \tag{25}$$

Since $E_t\left[w_{t+1}\right] = 0$, we obtain

$$\begin{aligned}
G_x^{(\tau)} &= \left(Z + G_x^{(\tau-1)}\right)\psi_x + \left(Z\phi_x - Z\right) \\
G_0^{(\tau)} &= G_0^{(\tau-1)} + Z\phi_q + \left(Z + G_x^{(\tau-1)}\right)\psi_q.
\end{aligned}$$

Under the subjective measure, the derivation is unchanged, except for the last line in (25), which now involves the subjective expectation $\widetilde{E}_t\left[w_{t+1}\right] = \overline{H} + HF\widehat{x}_t$. Then,

$$\begin{aligned}
\widetilde{G}_x^{(\tau)} &= \left(Z + \widetilde{G}_x^{(\tau-1)}\right)\psi_x + \left(Z\phi_x - Z\right) + \left(\left(Z + \widetilde{G}_x^{(\tau-1)}\right)\psi_w + Z\phi_w\right)HF \\
\widetilde{G}_0^{(\tau)} &= \widetilde{G}_0^{(\tau-1)} + Z\phi_q + \left(Z + \widetilde{G}_x^{(\tau-1)}\right)\psi_q + \left(\left(Z + \widetilde{G}_x^{(\tau-1)}\right)\psi_w + Z\phi_w\right)\overline{H}
\end{aligned}$$

Consequently,

$$\Delta_t^{(\tau)} = \left(\widetilde{G}_x^{(\tau)} - G_x^{(\tau)}\right)\widehat{x}_t + \widetilde{G}_0^{(\tau)} - G_0^{(\tau)}.$$

When the dynamics (24) are stationary and demeaned, $\overline{H}$, $\phi_q$, $\phi_x$, $\phi_w$, and $\phi_q$ are all zero, and we get explicit expressions

$$\begin{aligned}
G_x^{(\tau)} &= Z\left(\psi_x\right)^\tau \\
G_0^{(\tau)} &= Z\sum_{i=0}^{\tau-1}\left(\psi_x\right)^i\psi_q = Z\left(I - \psi_x\right)^{-1}\left(I - \left(\psi_x\right)^\tau\right)\psi_q \\
\widetilde{G}_x^{(\tau)} &= Z\left(\psi_x + \psi_w HF\right)^\tau \\
\widetilde{G}_0^{(\tau)} &= Z\sum_{i=0}^{\tau-1}\left(\psi_x + \psi_w HF\right)^i\psi_q = Z\left(I - \left(\psi_x + \psi_w HF\right)\right)^{-1}\left(I - \left(\psi_x + \psi_w HF\right)^\tau\right)\psi_q.
\end{aligned}$$

# B Linear approximation of models with robust preferences

The linear approximation in this paper builds on the series expansion method used in Holmes (1995), Lombardo (2010), and Borovička and Hansen (2014). The innovation in this paper consists of adapting the series expansion method to an approximation of models with robust preferences to derive a linear approximation that allows for endogenously determined time-varying belief distortions. The critical step in the expansion lies in the joint perturbation of the shock vector $w_t$ and the penalty process $\theta_t$.

## B.1 Law of motion

We start with the approximation of the model for the law of motion (4) with a sufficiently smooth policy rule $\psi$. We consider a class of models indexed by the scalar perturbation parameter $\mathsf{q}$ that scales the volatility of the shock vector $w_t$

$$x_{t+1}(\mathsf{q}) = \psi\left(x_t(\mathsf{q}), \mathsf{q}w_{t+1}, \mathsf{q}\right) \tag{26}$$

and assume that there exists a series expansion of $x_t$ around $\mathsf{q} = 0$:

$$x_t(\mathsf{q}) \approx \bar{x} + \mathsf{q}x_{1t} + \frac{\mathsf{q}^2}{2}x_{2t} + \ldots.$$

The processes $x_{jt}$, $j = 0, 1, \ldots$ can be viewed as derivatives of $x_t$ with respect to the perturbation parameter, and their laws of motion can be inferred by differentiating (26) $j$ times and evaluating the derivatives at $\mathsf{q} = 0$, assuming that $\psi$ is sufficiently smooth. Here, we focus only on the approximation up to the first order:

$$\begin{aligned} \bar{x} &= \psi(\bar{x}, 0, 0) \\ x_{1t+1} &= \psi_q + \psi_x x_{1t} + \psi_w w_{t+1}. \end{aligned} \tag{27}$$

We begin with a case in which the equilibrium dynamics of $x_t$ are stationary. Extensions to nonstationary environments are considered in Appendix B.7.

## B.2 Continuation values

We now focus on the expansion of the continuation value recursion. Substituting the belief distortion (5) into the recursion (2) yields

$$V_t = u(x_t) - \frac{\beta}{\theta_t}\log E_t\left[\exp\left(-\theta_t V_{t+1}\right)\right]. \tag{28}$$

We are looking for an expansion of the continuation value

$$V_t(\mathsf{q}) \approx \bar{V} + \mathsf{q}V_{1t}. \tag{29}$$

To derive the solution of the continuation value, we are interested in expanding the following perturbation of the recursion:

$$V_t(\mathsf{q}) = u\left(x_t(\mathsf{q}), \mathsf{q}\right) - \beta\frac{\mathsf{q}}{\overline{\theta}(\bar{x} + x_{1t})}\log E_t\left[\exp\left(-\frac{\overline{\theta}(\bar{x} + x_{1t})}{\mathsf{q}}V_{t+1}(\mathsf{q})\right)\right]. \tag{30}$$

Here, we utilized the fact that $\theta_t = \overline{\theta}x_t \approx \overline{\theta}(\bar{x} + x_{1t})$. More importantly, the perturbation scales jointly the volatility of the stochastic processes for $V_t$ and $u(x_t)$ with the magnitude of the penalty parameter $\theta_t$. In particular, the penalty parameter in the perturbation of equation (2) becomes $\mathsf{q}/\left[\overline{\theta}(\bar{x} + x_{1t})\right]$ and decreases jointly with the volatility of the shock process. This assumption will imply that the benchmark and subjective models do not converge as $\mathsf{q} \to 0$, and the linear approximation around a deterministic steady state yields a nontrivial contribution from the subjective dynamics.

Using the expansion of the period utility function

$$u\left(x_t(\mathsf{q}), \mathsf{q}\right) \approx \bar{u} + \mathsf{q}u_{1t} = \bar{u} + \mathsf{q}\left(u_x x_{1t} + u_q\right),$$

we get the deterministic steady-state (zeroth-order) term by setting $\mathsf{q} = 0$:

$$\bar{V} = (1 - \beta)^{-1}\bar{u}.$$

The first-order term in the expansion is derived by differentiating (30) with respect to $\mathsf{q}$ and is given by the recursion

$$V_{1t} = u_{1t} - \beta \frac{1}{\overline{\theta}\left(\bar{x} + x_{1t}\right)} \log E_t \left[\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_{1t+1}\right)\right]. \tag{31}$$

Since $\bar{x}$ is constant and $x_{1t}$ has linear dynamics (27), we hope to find linear dynamics for $V_{1t}$ as well. Since $u_t = u\left(x_t\right)$, we can make the guess that $V_t^i\left(\mathsf{q}\right) = V^i\left(x_t\left(\mathsf{q}\right), \mathsf{q}\right)$, which leads to the following expressions for the derivative of $V_t$:

$$V_{1t} = V_x x_{1t} + V_q.$$

Using this guess and comparing coefficients, equation (31) leads to a pair of algebraic equations for the unknown coefficients $V_x$ and $V_q$:

$$\begin{aligned} V_x &= u_x - \frac{\beta}{2} V_x \psi_w \psi_w' V_x' \overline{\theta} + \beta V_x \psi_x \\ V_q &= u_q - \frac{\beta}{2} \overline{\theta} \bar{x} V_x \psi_w \psi_w' V_x' + \beta V_x \psi_q + \beta V_q. \end{aligned}$$

The first from this pair of equations is a Riccati equation for $V_x$, which can be solved for given coefficients $\psi_x$ and $\psi_w$.

### B.3 Distortions and belief wedges

With the approximation of the continuation value at hand, we can derive the expansion of the one-period belief distortion $m_{t+1}$ that defines the subjective model relative to the benchmark model. As in (30), we scale the penalty parameter $\theta_t$ jointly with the volatility of the underlying shocks:

$$m_{t+1}\left(\mathsf{q}\right) = \frac{\exp\left(-\frac{1}{\mathsf{q}}\theta_t V_{t+1}\left(\mathsf{q}\right)\right)}{E_t\left[\exp\left(-\frac{1}{\mathsf{q}}\theta_t V_{t+1}\left(\mathsf{q}\right)\right)\right]} \approx m_{0,t+1} + \mathsf{q} m_{1,t+1}.$$

It turns out that in order to derive the correct first-order expansion, we are required to consider a second-order expansion of the continuation value

$$V_t\left(\mathsf{q}\right) \approx \bar{V} + \mathsf{q} V_{1t} + \frac{\mathsf{q}}{2} V_{2t},$$

although the term $V_{2t}$ will be inconsequential for subsequent analysis. Substituting in expression (29) and noting that $\bar{V}$ is a deterministic term, we can approximate $m_{t+1}$ with

$$m_{t+1}\left(\mathsf{q}\right) \approx \frac{\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right)\left(V_{1t+1} + \frac{\mathsf{q}}{2} V_{2t+1}\right)\right)}{E_t\left[\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right)\left(V_{1t+1} + \frac{\mathsf{q}}{2} V_{2t+1}\right)\right)\right]}.$$

Differentiating with respect to $\mathsf{q}$ and evaluating at $\mathsf{q} = 0$, we obtain the expansion

$$\begin{aligned} m_{0t+1} &= \frac{\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_{1t+1}\right)}{E_t\left[\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_{1t+1}\right)\right]} \\ m_{1t+1} &= -\frac{1}{2\overline{\theta}\left(\bar{x} + x_{1t}\right)} m_{0t+1}\left[V_{2t+1} - E_t\left[m_{0t+1} V_{2t+1}\right]\right]. \end{aligned} \tag{32}$$

This expansion is distinctly different from the standard polynomial expansion familiar from the perturbation literature. First, observe that $m_{0t+1}$ is not constant, as one would expect for a zeroth-order term, but nonlinear in $V_{1t+1}$. However, since $E_t[m_{0t+1}] = 1$, we can treat $m_{0t+1}$ as a change of measure that will adjust the distribution of shocks that are correlated with $m_{0t+1}$. We will show that with Gaussian shocks, we can still preserve tractability. Further notice that $E_t[m_{1t+1}] = 0$.

The linear structure of $V_{1t}$ also has an important implication for the subjective belief distortion constructed from $m_{0t+1}$. Substituting into (32) yields

$$m_{0t+1} = \frac{\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_x \psi_w w_{t+1}\right)}{E_t\left[\exp\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right) V_x \psi_w w_{t+1}\right)\right]}.$$

This implies that for a function $f(w_{t+1})$ with a shock vector $w_{t+1} \sim N(0, I)$, the first-order approximation is given by

$$\begin{aligned} \widetilde{E}_t\left[f\left(w_{t+1}\right)\right] &= E_t\left[m_{t+1} f\left(w_{t+1}\right)\right] \\ &\approx f_0\left(w_{t+1}\right) + E_t\left[m_{0t+1} f_1\left(w_{t+1}\right)\right]. \end{aligned} \tag{33}$$

The distortion generating the subjective measure $\widetilde{P}$ is therefore approximated by the zeroth-order term $m_{0t+1}$, and the vector $w_{t+1}$ has the following distribution:

$$w_{t+1} \sim N\left(-\overline{\theta}\left(\bar{x} + x_{1t}\right)\left(V_x \psi_w\right)', I_k\right). \tag{34}$$

The mean of the shock is therefore time varying and depends on the linear process $x_{1t}$.

It follows that the belief wedges for the one-period-ahead forecast of the vector of variables $x_t$ are given by

$$\Delta_t^{(1)} = \widetilde{E}_t\left[x_{t+1}\right] - E_t\left[x_{t+1}\right] = \psi_w \widetilde{E}_t\left[w_{t+1}\right] = -\overline{\theta}\left(\bar{x} + x_{1t}\right)\left(\psi_w \psi_w'\right) V_x'.$$

Belief wedges for longer-horizon forecasts are then computed using formulas from Appendix A, observing that we can set

$$F = \overline{\theta}, \qquad H = -\left(V_x \psi_w\right)', \qquad \overline{H} = -\overline{\theta}\bar{x}\left(V_x \psi_w\right)'.$$

The terms $\psi_w$ and $V_x$ are functions of structural parameters in the model solved in the following section.

### B.4 Equilibrium conditions

We assume that equilibrium conditions in our framework can be written as

$$0 = E_t\left[\widetilde{g}\left(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t\right)\right], \tag{35}$$

where $\widetilde{g}$ is an $n \times 1$ vector function and the dynamics for $x_t$ are implied by (4). This vector of equations includes expectational equations such as Euler equations of the robust household, which can be represented using subjective belief distortions $m_{t+1}$. We therefore assume that we can write the $j$-th component of $\widetilde{g}$ as

$$\widetilde{g}^j\left(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t\right) = m_{t+1}^{\sigma_j} g^j\left(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t\right),$$

where $\sigma_j \in \{0, 1\}$ captures whether the expectation in the $j$-th equation is under the household's subjective model. In particular, all nonexpectational equations and all equations not involving agents' preferences have $\sigma_j = 0$. System (35) can then be written as

$$0 = E_t\left[\mathbb{M}_{t+1} g\left(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t\right)\right], \tag{36}$$

where $\mathbb{M}_{t+1} = \text{diag}\left\{m_{t+1}^{\sigma_1}, \ldots, m_{t+1}^{\sigma_n}\right\}$ is a diagonal matrix of the belief distortions, and $g$ is independent of $\theta_t$. The zeroth-order and first-order expansions are

$$\begin{aligned}
0 &= E_t\left[\mathbb{M}_{0t+1}g_{0t+1}\right] = g_{0t+1} \\
0 &= E_t\left[\mathbb{M}_{0t+1}g_{1t+1}\right] + E_t\left[\mathbb{M}_{1t+1}g_{0t+1}\right] = E_t\left[\mathbb{M}_{0t+1}g_{1t+1}\right],
\end{aligned}$$

where the last equality follows from $E_t\left[m_{1t+1}\right] = 0$.

For the first-order derivative of the equilibrium conditions, we have

$$0 = E_t\left[\mathbb{M}_{0t+1}g_{1t+1}\right]. \tag{37}$$

The first-order term in the expansion of $g_{t+1}$ is given by

$$\begin{aligned}
g_{1t+1} &= g_{x+}x_{1t+1} + g_x x_{1t} + g_{x-}x_{1t-1} + g_{w+}w_{t+1} + g_w w_t + g_q = \\
&= \left[\left(g_{x+}\psi_x + g_x\right)\psi_x + g_{x-}\right]x_{1t-1} + \left[\left(g_{x+}\psi_x + g_x\right)\psi_w + g_w\right]w_t + \\
&\quad + \left(g_{x+}\psi_x + g_{x+} + g_x\right)\psi_q + g_q + \left(g_{x+}\psi_w + g_{w+}\right)w_{t+1},
\end{aligned} \tag{38}$$

where symbols $x_+, x, x_-, w_+, w, q$ represent partial derivatives with respect to $x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t$ and $q$, respectively. Given the subjective distribution of the shock vector (34), we can write

$$\widetilde{E}_t\left[w_{t+1}\right] = -\left(V_x\psi_w\right)'\overline{\theta}\left[\left(\bar{x} + \psi_q\right) + \psi_x x_{1t-1} + \psi_w w_t\right].$$

Let $[A]^i$ denote the $i$-th row of matrix $A$. Notice that

$$\left[g_{x+}\psi_w + g_{w+}\right]^i\left(V_x\psi_w\right)'\overline{\theta}$$

is a $1 \times n$ vector. Construct the $n \times n$ matrix $\mathbb{E}$ by stacking these row vectors for all equations $i = 1, \ldots, n$:

$$\mathbb{E} = \text{stack}\left\{\sigma_i\left[g_{x+}\psi_w + g_{w+}\right]^i\left(V_x\psi_w\right)'\overline{\theta}\right\},$$

which contains non-zero rows for expectational equations under the subjective model. Using matrix $\mathbb{E}$, we construct the conditional expectation of the last term in $g_{1t+1}$ in (38). In particular,

$$\begin{aligned}
0 &= E_t\left[\mathbb{M}_{0t+1}g_{1t+1}\right] = \\
&= \left[\left(g_{x+}\psi_x + g_x\right)\psi_x + g_{x-}\right]x_{1t-1} + \left[\left(g_{x+}\psi_x + g_x\right)\psi_w + g_w\right]w_t + \\
&\quad + \left(g_{x+}\psi_x + g_{x+} + g_x\right)\psi_q + g_q - \mathbb{E}\left[\left(\bar{x} + \psi_q\right) + \psi_x x_{1t-1} + \psi_w w_t\right].
\end{aligned}$$

Equation (37) is thus a system of linear second-order stochastic difference equations. There are well-known results that discuss the conditions under which there exists a unique stable equilibrium path to this system (Blanchard and Kahn (1980), Sims (2002)). We assume that such conditions are satisfied. Comparing coefficients on $x_{1t-1}$, $w_t$, and the constant term implies that

$$0 = \left(g_{x+}\psi_x + g_x - \mathbb{E}\right)\psi_x + g_{x-} \tag{39}$$

$$0 = \left(g_{x+}\psi_x + g_x - \mathbb{E}\right)\psi_w + g_w \tag{40}$$

$$0 = \left(g_{x+}\psi_x + g_{x+} + g_x\right)\psi_q + g_q - \mathbb{E}\left(\bar{x} + \psi_q\right). \tag{41}$$

These equations need to be solved for $\psi_x$, $\psi_w$, $\psi_q$, and $V_x$ where

$$V_x = u_x - \frac{\beta}{2} V_x \psi_w \psi_w' V_x' \overline{\theta} + \beta V_x \psi_x \tag{42}$$

and

$$\mathbb{E} = \text{stack} \left\{ \sigma_i \left[ g_{x+} \psi_w + g_{w+} \right]^i (V_x \psi_w)' \, \overline{\theta} \right\}. \tag{43}$$

### B.5 Multiple belief distortions

We proceeded with the derivation of the approximation under the assumption that there is only a single belief distortion affecting the equilibrium equations. This has been done for notational simplicity, and the extension to a framework with multiple agents endowed with heterogeneous belief distortions stemming from robust preferences is straightforward. Let us assume that there are $J$ agents with alternative belief distortions characterized by $\left( V_t^j, m_{t+1}^j, \overline{\theta}^j \right)$, $j = 1, \ldots J$. The system of equilibrium conditions (36) given by

$$0 = E_t \left[ \mathbb{M}_{t+1} g \left( x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t \right) \right]$$

with $\mathbb{M}_{t+1} = \text{diag} \left\{ m_{t+1}^{\sigma_1}, \ldots, m_{t+1}^{\sigma_n} \right\}$ can then be extended to include alternative belief distortions indexed by $\sigma_i \in \{0, 1, \ldots, J\}$ where $m_{t+1}^0 \equiv 1$ denotes an undistorted equation. Subsequently, there are $J$ distorted means of the innovations

$$\widetilde{E}_t^j \left[ w_{t+1} \right] = - \left( V_x^j \psi_w \right)' \overline{\theta}^j \left[ (\bar{x} + \psi_q) + \psi_x x_{1t-1} + \psi_w w_t \right]$$

that distort individual equations. Matrix $\mathbb{E}$ in (43) that collects the distortions of the equilibrium conditions then becomes

$$\mathbb{E} = \text{stack} \left\{ \left[ g_{x+} \psi_w + g_{w+} \right]^i \left( V_x^{\sigma_i} \psi_w \right)' \, \overline{\theta}^{\sigma_i} \right\},$$

where $\sigma_i = 0$ corresponds to no distortion, and hence the $i$-th row is a row of zeros. The structure of the system (39)–(43) remains the same except that we now have $J$ recursions for $V_x^j$ in (42) and a modified matrix $\mathbb{E}$.

### B.6 Special case: $\theta_t$ is an exogenous AR(1) process

In the application, we consider a special case that restricts $\theta_t$ to be an exogenous AR(1) process. With a slight abuse in notation, this restriction can be implemented by replacing the vector of variables $x_t$ with $(x_t', f_t)'$ where $f_t$ is a scalar AR(1) process representing the time variation in subjective beliefs as an exogenously specified shock:

$$f_{t+1} = (1 - \rho_f) \, \bar{f} + \rho_f f_t + \sigma_f w_{t+1}^f. \tag{44}$$

The dynamics of the model then satisfy

$$x_t = \psi \left( x_{t-1}, w_t, f_t \right) \tag{45}$$

with steady state $\left( \bar{x}', \bar{f} \right)'$. The vector $\overline{\theta}$ in (3) is then partitioned as $\overline{\theta}' = \left( \overline{\theta}_x', \overline{\theta}_f \right) = (0_{1 \times n-1}, 1)$, and thus $\theta_t = f_t$, as used in the specification in Section 4. Constructing the first-order series expansion of (45), we obtain

$$\begin{pmatrix} x_{1t+1} \\ f_{1t+1} \end{pmatrix} = \begin{pmatrix} \psi_q \\ 0 \end{pmatrix} + \begin{pmatrix} \psi_x & \rho_f \psi_{xf} \\ 0 & \rho_f \end{pmatrix} \begin{pmatrix} x_{1t} \\ f_{1t} \end{pmatrix} + \begin{pmatrix} \psi_w & \sigma_f \psi_{xf} \\ 0 & \sigma_f \end{pmatrix} \begin{pmatrix} w_{t+1} \\ w_{t+1}^f, \end{pmatrix}$$

where $w_{t+1}$ and $w_{t+1}^f$ are uncorrelated innovations. The matrices $\psi_x$ and $\psi_w$ thus do not involve any direct impact of the dynamics of the belief shock $f_{1t}$ and the matrix $\psi_{xf}$ captures how the dynamics of $f_{1t}$ influence the dynamics of endogenous state variables.

Let us further assume that the system (35) represents the equilibrium restrictions of the model except for equation (44). In this case, the function $g$ does not directly depend on $f$. Repeating the expansion of the equilibrium conditions from Section B.4 and comparing coefficients on $x_{t-1}$, $f_{t-1}$, $w_t$, and the constant term yields the set of conditions for matrices $\psi_x$, $\psi_w$, $\psi_{xf}$, and $\psi_q$:

$$\begin{aligned} 0 &= (g_{x+}\psi_x + g_x)\,\psi_x + g_{x-} && (46) \\ 0 &= (g_{x+}\rho_f\psi_{xf} - \mathbb{E}) + (g_{x+}\psi_x + g_x)\,\psi_{xf} && (47) \\ 0 &= (g_{x+}\psi_x + g_x)\,\psi_w + g_w && (48) \\ 0 &= (g_{x+}\psi_x + g_{x+} + g_x)\,\psi_q + g_q - \mathbb{E}\bar{f} && (49) \end{aligned}$$

with

$$\begin{aligned} V_x &= u_x + \beta V_x \psi_x && (50) \\ V_f &= u_f - \frac{\beta\overline{\theta}_f}{2}\left(V_f^2\sigma_f^2 + 2V_x\psi_{xf}\sigma_f^2 V_f + V_x\left(\sigma_f^2\psi_{xf}\psi_{xf}' + \psi_w\psi_w'\right)V_x'\right) && (51) \\ &\quad +\beta\left(V_f\rho_f + V_x\psi_{xf}\rho_f\right) \\ \mathbb{E} &= \text{stack}\left\{\sigma^i\left[g_{x+}\psi_{xf}\sigma_f^2\left(V_f + V_x\psi_{xf}\right) + \left(g_{x+}\psi_w + g_{w+}\right)\psi_w' V_x'\right]^i\right\}\overline{\theta}_f. && (52) \end{aligned}$$

This set of equations is the counterpart of equations (39)–(43) and can be solved sequentially. First, notice that equations (46) and (48) can be solved for $\psi_x$ and $\psi_w$, and these coefficients are not affected by the dynamics of $f_t$. But the equilibrium dynamics of $x_t$ are affected by movements in $f_t$ through the coefficient $\psi_{xf}$. The coefficient $\rho_f\psi_{xf}$ introduces an additional component in the time-varying drift of $x_t$, while $\sigma_f\psi_{xf}$ is an additional source of volatility arising from the shocks to household's subjective beliefs.

We solve this set of equations by backward induction. First, we use (39), (43), and (50) to find the rational expectations solution for $\psi_x$, $\psi_w$, $V_x$. Then we postulate that (45) is in fact a time-dependent law of motion

$$x_t = \psi^t\left(x_{t-1}, w_t, f_t\right)$$

with terminal condition at a distant date $T$

$$x_T = \psi^T\left(x_{T-1}, w_T, 0\right).$$

This corresponds to assuming that starting from date $T$, subjective belief distortions are absent in the model.

Plugging this guess into the set of equilibrium conditions, we obtain the set of algebraic equations

$$
\begin{aligned}
0 &= \left(g_{x+}\psi_{xf}^{t+1}\rho_f - \mathbb{E}^{t+1}\right) + \left(g_{x+}\psi_x + g_x\right)\psi_{xf}^{t} && (53)\\
V_f^t &= u_f - \frac{\beta\overline{\theta}_f}{2}\left(\left(V_f^{t+1}\sigma_f\right)^2 + 2V_x\psi_{xf}^{t+1}\sigma_f^2 V_f^{t+1} + V_x\left(\sigma_f^2\psi_{xf}^{t+1}\left(\psi_{xf}^{t+1}\right)' + \psi_w\psi_w'\right)V_x'\right) && (54)\\
&\quad + \beta\rho_f\left(V_f^{t+1} + V_x\psi_{xf}^{t+1}\right)\\
\mathbb{E}^{t+1} &= \left[g_{x+}\psi_{xf}^{t+1}\left(V_f^{t+1} + V_x\psi_{xf}^{t+1}\right)\sigma_f^2 + \left(g_{x+}\psi_w + g_{w+}\right)\psi_w' V_x'\right]\overline{\theta}_f. && (55)
\end{aligned}
$$

Equation (53) can then be solved for

$$
\psi_{xf}^{t} = \left(g_{x+}\psi_x + g_x\right)^{-1}\left(\mathbb{E}^{t+1} - g_{x+}\psi_{xf}^{t+1}\rho_f\right) \tag{56}
$$

Iterating backward on equations (54)–(56) until convergence yields the stationary solution of the economy with subjective beliefs as a long-horizon limit of an economy where these concerns vanish at a distant $T$. The system converges as long as its dynamics are stationary under the subjective model. Once we find the limit $\lim_{t\to-\infty}\mathbb{E}^t = \mathbb{E}$, we can also determine

$$
\psi_q = \left(g_{x+}\psi_x + g_{x+} + g_x\right)^{-1}\left(\mathbb{E}\bar{f} - g_q\right).
$$

### B.7 Nonstationary models

For the purpose of applying the expansion method, we assumed that the state vector $x_t$ is stationary. Our framework can, however, deal with deterministic or stochastic trends featured in macroeconomic models. Specifically, let us assume that there exists a vector-valued stochastic process $z_t$ such that the dynamics of $x_t$ can be written as

$$
\begin{aligned}
x_t &= \widehat{x}_t + z_t && (57)\\
z_{t+1} - z_t &= \phi\left(\widehat{x}_t, w_{t+1}\right),
\end{aligned}
$$

where $\widehat{x}_t$ is a stationary vector Markov process that replaces dynamics (4):

$$
\widehat{x}_{t+1} = \psi\left(\widehat{x}_t, w_{t+1}\right).
$$

The process $z_t$ thus has stationary increments and $x_t$ and $z_t$ are cointegrated, element by element. A typical example of an element in $z_t$ is a productivity process with a permanent component. Once we solve for the stationary dynamics of $\widehat{x}_t$, we can obtain the dynamics of $x_t$ in a straightforward way using (57).

Assume that the period utility function can be written in the form

$$
u\left(x_t\right) = \widehat{u}\left(\widehat{x}_t\right) + Z^u z_t,
$$

where $Z^u$ is a selection vector that selects the appropriate scaling from the vector $z_t$. For example,

$$
u\left(x_t\right) = \log C_t = \log\left[\widehat{C}_t\exp\left(Z^u z_t\right)\right] = \log\widehat{C}_t + Z^u z_t,
$$

where $Z^u z_t$ is the nonstationary component of the logarithm of consumption $\log C_t$, and $\widehat{C}_t = \widehat{C}\left(\widehat{x}_t\right)$ is the stationary part. It follows from equation (28) that we can write

$$V_t = \widehat{V}\left(\widehat{x}_t\right) + \left(1-\beta\right)^{-1} Z^u z_t,$$

and the stationary component of the continuation value $\widehat{V}\left(\widehat{x}_t\right)$ satisfies the recursion

$$\widehat{V}\left(\widehat{x}_t\right) = \widehat{u}\left(\widehat{x}_t\right) - \frac{\beta}{\theta_t}\log E_t\left[\exp\left(-\theta_t\left(\widehat{V}\left(\widehat{x}_{t+1}\right) + \left(1-\beta\right)^{-1} Z^u \phi\left(\widehat{x}_t, w_{t+1}\right)\right)\right)\right].$$

The first-order expansion of $\phi$ yields

$$\begin{aligned} \bar{z}_{t+1} - \bar{z}_t &= \phi\left(\bar{x}, 0\right) \\ z_{1t+1} - z_{1t} &= \phi_q + \phi_x \widehat{x}_{1t} + \phi_w w_{t+1}, \end{aligned}$$

where $\bar{x}$ is the steady state of $\widehat{x}_t$. We can now proceed as in the stationary case except using the expansion of functions $\widehat{u}$ and $\widehat{V}$. We have

$$\bar{V} = \left(1-\beta\right)^{-1}\left[\bar{u} + \beta\left(1-\beta\right)^{-1} Z^u \phi\left(\bar{x}, 0\right)\right]$$

and

$$\widehat{V}_{1t} = V_x \widehat{x}_{1t} + V_q$$

with

$$\begin{aligned} V_x &= u_x + \beta\left[V_x \psi_x + \left(1-\beta\right)^{-1} Z^u \phi_x\right] - \frac{\beta}{2}\left|V_x \psi_w + \left(1-\beta\right)^{-1} Z^u \phi_w\right|^2 \overline{\theta} \\ V_q &= u_q + \beta\left[V_q + V_x \psi_q + \left(1-\beta\right)^{-1} Z^u \phi_q\right] - \frac{\beta}{2}\overline{\theta}\bar{x}\left|V_x \psi_w + \left(1-\beta\right)^{-1} Z^u \phi_w\right|^2. \end{aligned}$$

The zeroth-order distortion is consequently given by

$$m_{0t+1} = \frac{\exp\left(-\overline{\theta}\left(\bar{x} + \widehat{x}_{1t}\right)\left(V_x \psi_w + \left(1-\beta\right)^{-1} Z^u \phi_w\right) w_{t+1}\right)}{E_t\left[\exp\left(-\overline{\theta}\left(\bar{x} + \widehat{x}_{1t}\right)\left(V_x \psi_w + \left(1-\beta\right)^{-1} Z^u \phi_w\right) w_{t+1}\right)\right]}$$

so that under the subjective belief,

$$w_{t+1} \sim N\left(-\overline{\theta}\left(\bar{x} + \widehat{x}_{1t}\right)\left(V_x \psi_w + \left(1-\beta\right)^{-1} Z^u \phi_w\right)', I_k\right).$$

Equation (17) then becomes

$$\begin{aligned} \widehat{x}_{1t+1} &= \psi_q - \overline{\theta}\bar{x}\psi_w\left(V_x \psi_w + \left(1-\beta\right)^{-1} Z^u \phi_w\right)' \\ &\quad + \left[\psi_x - \psi_w\left(V_x \psi_w + \left(1-\beta\right)^{-1} Z^u \phi_w\right)' \overline{\theta}\right]\widehat{x}_{1t} + \psi_w \widetilde{w}_{t+1} \\ &= \widetilde{\psi}_q + \widetilde{\psi}_x x_{1t} + \psi_w \widetilde{w}_{t+1}. \end{aligned}$$

Comparing these dynamics under the subjective belief with those under the data-generating process, we can again construct belief wedges for longer-horizon forecasts as in Section B.3. Under the nonstationary dynamics, these wedges $\Delta_t^{(j)} = \widetilde{E}_t\left[x_{t+j}\right] - E_t\left[x_{t+j}\right]$ are computed using the recursive calculations outlined in Appendix A, imposing

$$
\begin{aligned}
F &= \overline{\theta} \\
H &= -\left(V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right)' \\
\overline{H} &= -\left(\overline{\theta}\bar{x}\right)\left(V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right)'.
\end{aligned}
\tag{58}
$$

To solve for the equilibrium dynamics, notice that we are still solving the set of equations (39)–(41) but now with $V_x$ and $\mathbb{E}$ given by

$$
\begin{aligned}
V_x &= u_x + \beta\left[V_x\psi_x + (1-\beta)^{-1} Z^u\phi_x\right] - \frac{\beta}{2}\left|V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right|^2\overline{\theta} \\
\mathbb{E} &= \text{stack}\left\{\sigma_i\left[g_{x+}\psi_w + g_{w+}\right]^i\left(V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right)'\overline{\theta}\right\}.
\end{aligned}
$$

In the special case described in Section B.6, the belief shock $f_t$ is modeled as an exogenous AR(1) process. The first-order dynamics of the stochastic growth rate can be expressed as

$$
z_{1t+1} - z_{1t} = \phi_q + \phi_x\widehat{x}_{1t} + \phi_{xf}f_{1t} + \phi_w w_{t+1} + \phi_{wf}w^f_{t+1}.
$$

The only modifications appearing in the model solution are those related to the continuation value recursion and the shock distortion in $\mathbb{E}$. Specifically,

$$
\begin{aligned}
V_x &= u_x + \beta\left[V_x\psi_x + (1-\beta)^{-1} Z^u\phi_x\right] \\
V_f &= u_f + \beta\left(\rho_f V_f + \rho_f V_x\psi_{xf} + (1-\beta)^{-1} Z^u\phi_{xf}\right) \\
&\quad -\frac{\beta\overline{\theta}_f}{2}\left|V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right|^2 - \frac{\beta\overline{\theta}_f}{2}\left|V_x\psi_{xf}\sigma_f + V_f\sigma_f + (1-\beta)^{-1} Z^u\phi_{wf}\right|^2 \\
\mathbb{E} &= \text{stack}\left\{\sigma^i\left[\left(g_{x+}\psi_w + g_{w+}\right)\left(V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right)'\right]^i\right\}\overline{\theta}_f \\
&\quad +\text{stack}\left\{\sigma^i\left[g_{x+}\psi_{xf}\sigma_f\left(V_f\sigma_f + V_x\psi_{xf}\sigma_f + (1-\beta)^{-1} Z^u\phi_{wf}\right)\right]^i\right\}\overline{\theta}_f.
\end{aligned}
$$

In the recursive form, $V_f$ and $\mathbb{E}$ can be solved by iterating on the pair of equations

$$
\begin{aligned}
V_f^t &= u_f + \beta\left(\rho_f V_f^{t+1} + \rho_f V_x\psi_{xf}^{t+1} + (1-\beta)^{-1} Z^u\phi_{xf}\right) \\
&\quad -\frac{\beta\overline{\theta}_f}{2}\left|V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right|^2 - \frac{\beta\overline{\theta}_f}{2}\left|V_x\psi_{xf}^{t+1}\sigma_f + V_f^{t+1}\sigma_f + (1-\beta)^{-1} Z^u\phi_{wf}\right|^2 \\
\mathbb{E}^{t+1} &= \text{stack}\left\{\sigma^i\left[\left(g_{x+}\psi_w + g_{w+}\right)\left(V_x\psi_w + (1-\beta)^{-1} Z^u\phi_w\right)'\right]^i\right\}\overline{\theta}_f \\
&\quad +\text{stack}\left\{\sigma^i\left[g_{x+}\psi_{xf}^{t+1}\sigma_f\left(V_f^{t+1}\sigma_f + V_x\psi_{xf}^{t+1}\sigma_f + (1-\beta)^{-1} Z^u\phi_{wf}\right)\right]^i\right\}\overline{\theta}_f
\end{aligned}
$$

together with equation (56), which remains unchanged.

## C Sequence problem formulation and penalty functions

We postulated the subjective belief problem in Section 3 directly in its recursive form. In order to formulate the sequence problem, define $M_{t,t+j} = \Pi_{k=1}^{j} m_{t+k}$, and the penalty function

$$\mathcal{E}_t \doteq E_t \sum_{j=0}^{\infty} \beta^j \left[ M_{t,t+j} \frac{\beta}{\theta_{t+j}} E_{t+j} \left[ m_{t+j+1} \log m_{t+j+1} \right] \right]. \tag{59}$$

This penalty function has the recursive representation

$$\mathcal{E}_t = \frac{\beta}{\theta_t} E_t \left[ m_{t+1} \log m_{t+1} \right] + \beta E_t \left[ m_{t+1} \mathcal{E}_{t+1} \right].$$

Then, the sequence problem can be written as

$$V_t^* = \max_{\{y_{t+j}\}_{j=0}^{\infty}} \min_{\substack{\{m_{t+j}>0\}_{j=1}^{\infty} \\ E_{t+j-1}[m_{t+j}]=1}} \sum_{j=0}^{\infty} \beta^j E_t \left[ M_{t,t+j} u_{t+j} \right] + \mathcal{E}_t \tag{60}$$

where $y_t$ represents the vector of choice variables the period utility sequence $\{u_{t+j}\}_{j=0}^{\infty}$ implicitly depends on (these choice variables are omitted for simplicity from formulation (2)). This notation assumes that $\{\theta_{t+j}\}_{j=0}^{\infty}$ is positive, otherwise we replace minimization with maximization in states where $\theta_{t+j}$ is negative. Associated with the problem is a set of constraints on the choice variables analogous to that in (4).

The first-order conditions with respect to $m_{t+1+k}$, $k \geq 0$, yield

$$\begin{aligned} 0 \quad = \quad & \sum_{j=k+1}^{\infty} \beta^j M_{t,t+k} E_{t+1+k} \left[ M_{t+1+k,t+j} u_{t+j} \right] \\ & + \sum_{j=k+1}^{\infty} \beta^j M_{t,t+k} E_{t+1+k} \left[ M_{t+1+k,t+j} \frac{\beta}{\theta_{t+j}} E_{t+j} \left[ m_{t+j+1} \log m_{t+j+1} \right] \right] \\ & + \beta^k M_{t,t+k} \left[ \frac{\beta}{\theta_{t+k}} \left[ \log m_{t+1+k} + 1 \right] - \kappa_{t+k} \right] \end{aligned}$$

where $\beta^k M_{t,t+k} \kappa_{t+k}$ is the Lagrange multiplier on the constraint $E_{t+k} \left[ m_{t+1+k} \right] = 1$. The normalization implied by the Lagrange multiplier yields

$$m_{t+1+k} = \frac{\exp\left(-\theta_{t+k} V_{t+1+k}^*\right)}{E_{t+k} \left[ \exp\left(-\theta_{t+k} V_{t+1+k}^*\right) \right]}.$$

Suppose that the utility function and associated constraints for the maximization part of the problem in (60) satisfy standard properties yielding a strictly concave objective on a convex choice set, and, as in our application, $\theta_t$ does not depend on individual choices. Given the strict convexity of the entropy function, the minimax inequality is an equality, and the min and max operators in (60) can be exchanged. The specification of the penalty function and the principle of optimality also imply that we can write the value function recursively as

$$V_t^* = \max_{y_t} \min_{\substack{m_{t+1}>0 \\ E_t[m_{t+1}]=0}} u_t + \frac{\beta}{\theta_t} E_t \left[ m_{t+1} \log m_{t+1} \right] + E_t \left[ m_{t+1} V_{t+1}^* \right].$$

The penalty $\mathcal{E}_t$ in (59) represents the discounted average of future one-period-ahead entropies, weighted by the future penalty parameters $\theta_{t+j}$. In particular, it takes into account agent's anticipation of future variation in the degree of pessimism, reflected in the variation in $\theta_{t+j}$. This anticipation assure dynamic consistency of agent's beliefs and decisions. A similar penalty consisting of discounted average of future one-period ahead entropies, but averaged under the data-generating measure, has been used in Woodford (2010) and Adam and Woodford (2012), with a constant penalty parameter.

The penalty $\mathcal{E}_t$ is not the same as the infinite-horizon discounted entropy from Hansen and Sargent (2001a,b). This discounted entropy can be written as

$$\widetilde{\mathcal{E}}_t = \frac{\beta(1-\beta)}{\theta_t} E_t \sum_{j=0}^{\infty} \beta^j \left[M_{t,t+j} \log M_{t,t+j}\right],$$

with a recursive representation given by

$$\widetilde{\mathcal{E}}_t = \frac{\beta}{\theta_t} E_t \left[m_{t+1} \log m_{t+1}\right] + \beta E_t \left[\frac{\theta_{t+1}}{\theta_t} m_{t+1} \widetilde{\mathcal{E}}_{t+1}\right].$$

The value function defined as

$$\widetilde{V}_t^* = \max_{\{y_{t+j}\}_{j=0}^{\infty}} \min_{\substack{\{m_{t+j}\}_{j=1}^{\infty} \\ E_{t+j-1}[m_{t+j}]=1}} \sum_{j=0}^{\infty} \beta^j E_t \left[M_{t,t+j} u_{t+j}\right] + \widetilde{\mathcal{E}}_t,$$

then we can write $\widetilde{V}_t^*$ as

$$\begin{aligned}
\widetilde{V}_t^* \quad = \quad & \max_{\{y_{t+j}\}_{j=0}^{\infty}} \min_{\substack{\{m_{t+j}\}_{j=1}^{\infty} \\ E_{t+j-1}[m_{t+j}]=1}} u_t + \frac{\beta}{\theta_t} E_t \left[m_{t+1} \log m_{t+1}\right] \\
& + \beta E_t \left[ m_{t+1} \left( E_{t+1} \sum_{j=0}^{\infty} \beta^j \left[M_{t+1,t+1+j} u_{t+1+j}\right] + \frac{\theta_{t+1}}{\theta_t} \widetilde{\mathcal{E}}_{t+1} \right) \right].
\end{aligned}$$

This formulation is not recursive and hence leads to a dynamically inconsistent choice of belief distortions. The lack of dynamic consistency could be rectified by introducing commitment with promise-keeping constraints as in the constraint problems in Hansen et al. (2006) but the outcome would lead to a different set of belief distortions that does not take into account future variation in $\theta_{t+j}$.

## D Data and further empirical evidence

Macroeconomic data are collected from the Federal Reserve Bank of St. Louis database (FRED).[33] The data on households' expectations are obtained from the University of Michigan Surveys of Consumers.[34] We also use data from the Survey of Consumer Expectations administered by the Federal Reserve Bank of New York,[35] and data from the Survey of Professional Forecasters collected from the Federal Reserve Bank of

[33] Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org/.

[34] Surveys of Consumers, University of Michigan, http://www.sca.isr.umich.edu/. See also Thomas (1999) for details on the survey methodology.

[35] Survey of Consumer Expectations, Center for Microeconomic Data, Federal Reserve Bank of New York, https://www.newyorkfed.org/microeconomics/sce.

Philadelphia website.[36] See Table 4 for details.

We use the consumer price index for all urban consumers: all items (CPIAUSCL in FRED) to compute the rate of inflation in the data. Computing the belief wedges using the personal consumption expenditures (PCE) index from the Bureau of Economic Analysis as an alternative (PCEPI in FRED) would leave the cyclical component of the inflation wedge almost unchanged because the two series are highly correlated (correlation over the sample period is 0.95). However, the PCE series has a substantially lower mean (by 0.36% annually between 1982 and 2019), so using the PCE index as observations from the data-generating process would make households appear to overestimate inflation significantly more than in the case of the CPI. We prefer the CPI because its weighting is based on surveys of the composition of households' purchases, and is based on out-of-pocket expenditures, which are arguably more salient for the formation of households' beliefs.

For the rate of unemployment, we use the civilian unemployment rate (UNRATE in FRED) as the data counterpart. Since households in the Michigan Survey are asked about the *change* in the rate of unemployment, the potential issue with different average levels of alternative measures of unemployment that households could envision becomes irrelevant. We construct the level forecast as the realized UNRATE measure in the month when the forecast is made, plus the forecasted change in the unemployment rate from the Michigan Survey.

## D.1 Survey data

For the inflation rate in the Michigan Survey, we record the cross-sectional mean, median, and quartile answers. The survey question on the unemployment rate only records up/same/down responses. We use the method from Carlson and Parkin (1975) and Mankiw et al. (2003) to fit a time series of normal distributions to these qualitative responses. Let $q_t^u$, $q_t^s$, and $q_t^d$ be the fractions of survey answers up, same, down, respectively, recorded at time $t$. We assume that these categories are constructed from a continuous cross-sectional distribution of responses with normal density $N\left(\mu_t, \sigma_t^2\right)$. In particular, there exists a response threshold $a$ such that an answer on the interval $[-a, a]$ is recorded as "same". This implies

$$q_t^d = \Phi\left(\frac{-a-\mu_t}{\sigma_t}\right) \qquad q_t^u = 1 - \Phi\left(\frac{a-\mu_t}{\sigma_t}\right),$$

and thus

$$-a-\mu_t = \sigma_t \Phi^{-1}\left(q_t^d\right) \qquad a-\mu_t = \sigma_t \Phi^{-1}\left(1-q_t^u\right),$$

and therefore

$$\begin{aligned} \sigma_t &= \frac{2a}{\Phi^{-1}\left(1-q_t^u\right) - \Phi^{-1}\left(q_t^d\right)} \\ \mu_t &= a - \sigma_t \Phi^{-1}\left(1-q_t^u\right). \end{aligned}$$

The constant $a$ is then determined so that the time-series average of the cross-sectional dispersions $\sigma_t$ divided by the observed average cross-sectional dispersion for the SPF forecast corresponds to the analogous ratio for the inflation responses, for which we have dispersion data readily available. We use the obtained means $\mu_t$ as the time series of mean forecasts for the change in unemployment.

To verify that the obtained time series $\mu_t$ provides a meaningful fit to the actual mean forecast, we verify

[36]Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia, https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/.

**Households' expectations (Michigan Survey)**

| | |
|---|---|
| $\widetilde{E}_t\left[\sum_{j=1}^{4}\pi_{t+j}\right]$ | Expected change in prices during the next year (Table 32, variable PX1), mean and median responses and quartiles of the cross-sectional distribution of individual answers. Questions: "*During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?*" and "*By about what percent do you expect prices to go up, on the average, during the next 12 months?*" |
| $\widetilde{E}_t\left[\frac{1}{n}\sum_{j=1}^{4}u_{t+j}\right]$ | Expected unemployment rate during the next year (Table 30, variable UMEX), construction of mean response and the dispersion detailed in the text. Question: "*How about people out of work during the coming 12 months – do you think there will be more unemployment than now, about the same, or less?*" We also report results interpreting answers to this question as expected unemployment in one year, $\widetilde{E}_t\left[u_{t+4}\right]$. |

**Households' expectations (SCE)**

| | |
|---|---|
| $\widetilde{E}_t\left[\sum_{j=1}^{4}\pi_{t+j}\right]$ | Median one-year-ahead expected inflation rate (used in Figure 3). The time series is constructed by aggregating probabilistic responses to the question: "*In your view, what would you say is the percent chance that, over the next 12 months... the rate of inflation will be between $x_i$% and $x_{i+1}$%*" for a range of brackets across individual households. See Armantier et al. (2017) for details. |
| $\widetilde{P}_t[u_{t+4}]$ | Probability of unemployment being higher in one year than today (used in Figure 3). Mean response to the question: "*What do you think is the percent chance that 12 months from now the unemployment rate in the U.S. will be higher than it is now?*" |

**Survey of Professional Forecasters**

| | |
|---|---|
| $E_t\left[\sum_{j=1}^{4}\pi_{t+j}\right]$ | Forecasted CPI inflation rate, seasonally adjusted (CPI). Forecast at time $t$ is constructed as the mean survey forecast made in second month of quarter $t+1$, for CPI inflation rate between quarters $t$ and $t+4$. |
| $E_t[\frac{1}{n}\sum_{j=1}^{4}u_{t+j}]$ | Forecasted unemployment rate, seasonally adjusted (UNEMP). Forecast at time $t$ is constructed as the mean survey forecast made in second month of quarter $t+1$, for the average unemployment rate in quarters $t+1$ to $t+4$. |

**Macroeconomic variables (FRED)**

| | |
|---|---|
| $\pi_t$ | Consumer price index for all urban consumers: all items, seasonally adjusted (CPIAUCSL). Quarterly logarithmic growth rate, last month to last month of quarter. |
| $u_t$ | Civilian unemployment rate, quarterly, seasonally adjusted (UNRATE). |
| $\log\left(\mathcal{Y}_t/\mathcal{Y}_{t-1}\right)$ | Real gross domestic product, quarterly, seasonally adjusted annual rate (GDPC1). Quarterly logarithmic growth rate. |
| $\log\left(\mathcal{Y}_t/\bar{\mathcal{Y}}_t\right)$ | Output gap. Difference between real gross domestic product, quarterly, seasonally adjusted annual rate (GDPC1) and real potential output (GDPPOT). |

**Table 4:** Data definitions for key macroeconomic and survey variables.

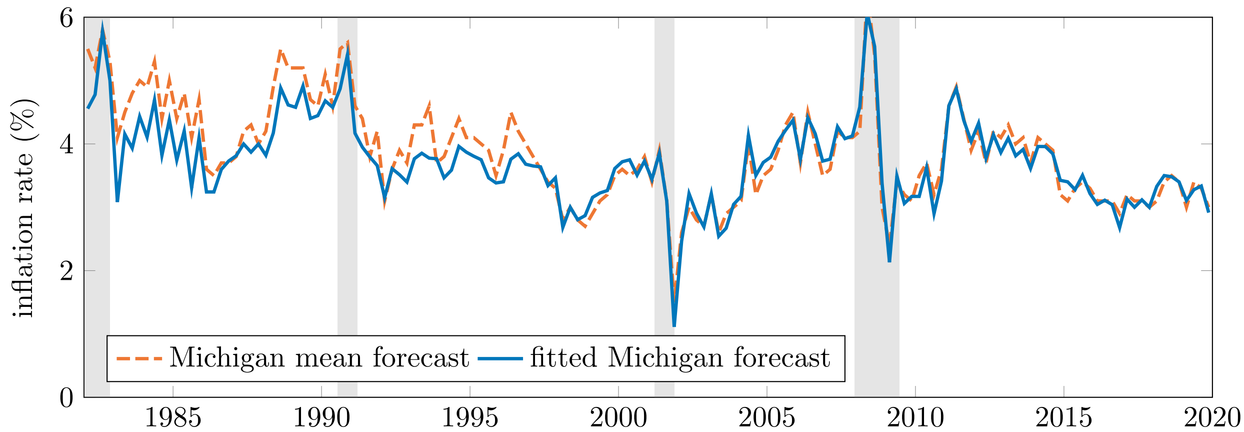


**Figure 15:** Fitted mean forecast of one-year-ahead inflation rate (solid line) constructed using the Carlson and Parkin (1975) and Mankiw et al. (2003) method from categorical data, and actual mean forecast in the Michigan Survey (dashed line). NBER recessions are shaded.

the methodology using the inflation forecast data. We categorize individual numerical inflation forecast responses in each period into three bins, $< 3\%$, $3 - 4\%$, and $\geq 5\%$, and then fit a time series of normal distributions as described above, using the three time series of answer shares in each of the bins as input. Figure 15 compares the time series of actual mean forecasts with the time series of fitted means constructed using categorical data. The correlation between the two series is 93.3%, and the time-series averages differ only by 0.15%, providing strong support for the methodology as a plausible approximation of the actual mean forecast.

### D.2 Information sets

The construction of belief wedges requires taking a stance on how to align information sets available to surveyed households and the econometrician. We use a quarterly VAR, described in more detail in the next subsection, for our benchmark forecast under the data-generating (rational) measure. We use monthly data from the Michigan Survey and available micro data from the monthly cross sections of the survey for the period 1982Q1–2019Q4 in the main text. When computing the belief wedges relative to the VAR forecast, we use responses from the first month of quarter $t+1$ as those made by households with information available at the end of quarter $t$. Time-series moments for the wedges in this sample are summarized in Panel A of Table 5. The forecasting horizon for one-year-ahead forecasts is assumed to span quarters $t+1$ to $t+4$.

The Michigan Survey also contains aggregated data at the quarterly frequency starting from 1960. We use these quarterly time series for the time period 1960Q3–2019Q4 in Table 5. We use the responses reported during quarter $t+1$ as those made with information available to the households at the end of quarter $t$.

The SPF is administered during the second month of each quarter. To compute the belief wedge relative to the SPF forecast, we therefore use Michigan Survey responses from the second month of each quarter as well to align information sets for the two forecasts. We again use the benchmark time period 1982Q1–2019Q4. Forecasts made in the second month of quarter $t+1$ are assumed to span quarters $t+1$ to $t+4$ in the quarterly analysis. Panel C of Table 5 summarizes the data.

## D.3 Forecasting VAR

We use a standard quarterly forecasting VAR to compute the forecasts of inflation and unemployment under the data-generating measure. All time series are downloaded from FRED for the period 1960Q1–2019Q4: CPI inflation (CPIAUCSL, percentage change to a year ago), real GDP (GDPC1, annualized percentage quarterly change), unemployment rate (UNRATE), log change in the relative price of investment goods (PIRIC), capital utilization rate (CUMFNS), log hours worked per capita (average hours per worker PRS85006023 multiplied by the employment-population ratio CE16OV/CNP16OV), consumption rate of nondurables and services ((PCEND+PCESV)/GDP), investment rate (GPDI/GDP), and the federal funds rate (FEDFUNDS). The VAR is estimated with two lags. These choices for the forecasting VAR are similar to those made in Christiano et al. (2005), Del Negro et al. (2007), Christiano et al. (2011), or Christiano et al. (2016). We experimented with increasing the lag length and including additional forecasting variables, without materially affecting the results. Below we present the results for two such alternative VAR specifications from the literature. All VAR models are estimated in unrestricted form using linear regression.

## D.4 Further time-series evidence on the belief wedges

In the main text, we use belief wedges constructed using the Michigan Survey responses for the period 1982Q1–2019Q4. In Table 5, we quantify their comovement with the business cycle and provide alternative specifications for the belief wedges as well as alternative time periods as robustness checks.

For the inflation wedge, we show results for both the mean and median inflation forecast for the Michigan Survey. For the unemployment wedge, we produce two wedges based on alternative interpretations of the relevant question in the Michigan Survey. The wedge $\Delta_t^{(4)}(u)$ is the wedge for the forecast of the unemployment rate four quarters ahead. The wedge $\overline{\Delta}_t^{(4)}(u)$ is the wedge for the forecast of the average unemployment rate during the next four quarters.

Panel A in Table 5 contains time-series characteristics of the belief wedges for the benchmark case, constructed using survey data for the period 1982Q1–2019Q4, net of the corresponding VAR forecasts. This is our preferred time period because the Michigan Survey for this period contains better-quality disaggregated survey data at the monthly frequency that allow us to better align information sets (Appendix D.2), study the cross-sectional patterns between the belief wedges, and compare the Michigan Survey responses with available SPF forecasts.

The subsequent panels of Table 5 contain a variety of robustness checks. First, to ensure that our results are not driven by the Great Recession episode, we compute the moments excluding observations for the period 2007Q4–2009Q2. These results are reported in Panel B of Table 5. The correlation between the unemployment and inflation belief wedges remains positive, albeit somewhat smaller than in the full sample. In addition, the belief wedges retain a strong negative correlation with the business cycle.

Next, we use the Michigan Survey responses aggregated at the quarterly frequency that are available for a longer period 1960Q3–2019Q4. These results are reported in Panel C of Table 5. The patterns in the data are largely unchanged (information on the median inflation forecast is not available in the Michigan Survey for this time period). The belief wedges continue to be large, volatile, and countercyclical. The mean inflation wedge is somewhat smaller than in Panel A, and the lower correlation between the output gap and GDP growth implies that the wedges continue to be strongly countercyclical when using the output gap as the measure of economic activity, but the relationship with GDP growth is weaker.

As an alternative, we also construct the belief wedges using the responses from the Survey of Professional Forecasters as a measure of forecasts under the data-generating measure. Panel D from Table 5 provides

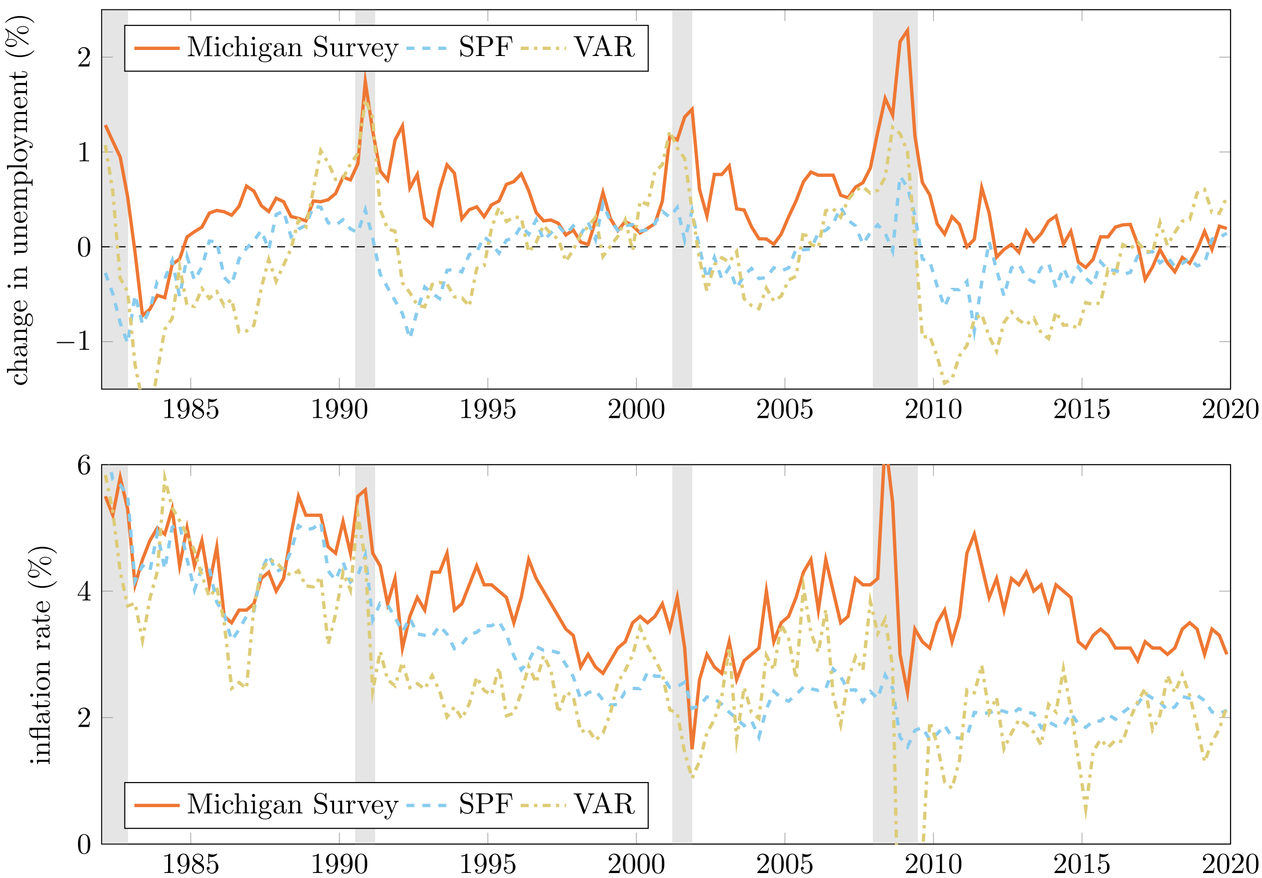


**Figure 16:** Comparison of mean forecasts from the Michigan Survey, Survey of Professional Forecasters, and the forecast obtained using the VAR. *Top panel:* Forecasted one-year-ahead change in unemployment. *Bottom panel:* Forecasted one-year-ahead inflation rate. NBER recessions are shaded.

the time-series characteristic for these wedges. As in the previous cases, we obtain large and volatile belief wedges that are highly negatively correlated with the business cycle.

In order to investigate the source of the belief wedges in more depth, we plot actual forecasts from the Michigan Survey, the Survey of Professional Forecasters, and the VAR in Figure 16. Since the Michigan Survey unemployment forecast is constructed for the change in unemployment rate, we convert the SPF and VAR forecasts by subtracting the contemporaneous unemployment rate. The top panel shows the forecasts of the change in unemployment rate, while the bottom panel shows the inflation rate forecasts.

The source of the fluctuations in the unemployment belief wedge can be broadly attributed to two factors. First, the Michigan Survey forecasts increase more aggressively during recessions than the SPF and VAR forecasts, and second, the Michigan Survey forecasts tend to decline more slowly after recessions end. Households in the Michigan Survey on average also only rarely predict that unemployment will decrease, which yields the upward average bias in the forecast. A similar pattern emerges for the inflation forecasts. Particularly notable is the large wedge between the forecasts in the post-Great Recession period.

As a robustness check of the VAR specification, we also present belief wedged constructed using two alternative VAR specifications for the rational forecast. Specifically, we use the specification from Del Negro et al. (2007) (DSSW), and from Christiano et al. (2011) (CTW), which is also used in Christiano et al.

| Panel A: 1982Q1–2019Q4, VAR forecast | | | correlation matrix | | | | | |
|---|---|---|---|---|---|---|---|---|
| | mean | std | (1) | (2) | (3) | (4) | (5) | (6) |
| (1) Unemployment wedge $\Delta_t^{(4)}(u)$ | 0.52 | 0.57 | 1.00 | 0.90 | 0.34 | 0.29 | −0.51 | −0.28 |
| (2) Unemployment wedge $\overline{\Delta}_t^{(4)}(u)$ | 0.48 | 0.48 | | 1.00 | 0.29 | 0.26 | −0.26 | −0.39 |
| (3) Mean inflation wedge $\Delta_t^{(4)}(\pi)$ | 1.22 | 0.97 | | | 1.00 | 0.93 | −0.29 | −0.49 |
| (4) Median inflation wedge $\Delta_t^{(4)}(\pi)$ | 0.44 | 1.06 | | | | 1.00 | −0.19 | −0.54 |
| (5) Output gap $\log\left(\mathcal{Y}_t/\bar{\mathcal{Y}}_t\right)$ | −1.53 | 1.98 | | | | | 1.00 | 0.46 |
| (6) GDP growth $\log\left(\mathcal{Y}_t/\mathcal{Y}_{t-4}\right)$ | 2.00 | 1.51 | | | | | | 1.00 |

| Panel B: 1982Q1–2019Q4, VAR forecast | | | correlation matrix | | | | | |
|---|---|---|---|---|---|---|---|---|
| (excluding 2007Q4–2009Q2) | mean | std | (1) | (2) | (3) | (4) | (5) | (6) |
| (1) Unemployment wedge $\Delta_t^{(4)}(u)$ | 0.49 | 0.55 | 1.00 | 0.90 | 0.22 | 0.15 | −0.50 | −0.17 |
| (2) Unemployment wedge $\overline{\Delta}_t^{(4)}(u)$ | 0.44 | 0.44 | | 1.00 | 0.11 | 0.07 | −0.25 | −0.27 |
| (3) Mean inflation wedge $\Delta_t^{(4)}(\pi)$ | 1.13 | 0.77 | | | 1.00 | 0.90 | −0.23 | −0.26 |
| (4) Median inflation wedge $\Delta_t^{(4)}(\pi)$ | 0.34 | 0.85 | | | | 1.00 | −0.10 | −0.34 |
| (5) Output gap $\log\left(\mathcal{Y}_t/\bar{\mathcal{Y}}_t\right)$ | −1.61 | 1.97 | | | | | 1.00 | 0.44 |
| (6) GDP growth $\log\left(\mathcal{Y}_t/\mathcal{Y}_{t-4}\right)$ | 2.14 | 1.32 | | | | | | 1.00 |

| Panel C: 1960Q3–2019Q4, VAR forecast | | | correlation matrix | | | | | |
|---|---|---|---|---|---|---|---|---|
| | mean | std | (1) | (2) | (3) | (4) | (5) | (6) |
| (1) Unemployment wedge $\Delta_t^{(4)}(u)$ | 0.40 | 0.61 | 1.00 | 0.88 | 0.24 | — | −0.31 | 0.01 |
| (2) Unemployment wedge $\overline{\Delta}_t^{(4)}(u)$ | 0.39 | 0.52 | | 1.00 | 0.19 | — | −0.12 | −0.14 |
| (3) Mean inflation wedge $\Delta_t^{(4)}(\pi)$ | 0.82 | 1.11 | | | 1.00 | — | −0.34 | −0.37 |
| (4) Median inflation wedge $\Delta_t^{(4)}(\pi)$ | — | — | | | | — | — | — |
| (5) Output gap $\log\left(\mathcal{Y}_t/\bar{\mathcal{Y}}_t\right)$ | −0.88 | 2.33 | | | | | 1.00 | 0.52 |
| (6) GDP growth $\log\left(\mathcal{Y}_t/\mathcal{Y}_{t-4}\right)$ | 2.23 | 1.78 | | | | | | 1.00 |

| Panel D: 1982Q1–2019Q4, SPF forecast | | | correlation matrix | | | | | |
|---|---|---|---|---|---|---|---|---|
| | mean | std | (1) | (2) | (3) | (4) | (5) | (6) |
| (1) Unemployment wedge $\Delta_t^{(4)}(u)$ | 0.49 | 0.49 | 1.00 | 0.97 | 0.18 | 0.12 | −0.34 | −0.54 |
| (2) Unemployment wedge $\overline{\Delta}_t^{(4)}(u)$ | 0.44 | 0.47 | | 1.00 | 0.16 | 0.12 | −0.15 | −0.50 |
| (3) Mean inflation wedge $\Delta_t^{(4)}(\pi)$ | 1.06 | 0.81 | | | 1.00 | 0.89 | −0.11 | −0.24 |
| (4) Median inflation wedge $\Delta_t^{(4)}(\pi)$ | 0.24 | 0.91 | | | | 1.00 | 0.01 | −0.30 |
| (5) Output gap $\log\left(\mathcal{Y}_t/\bar{\mathcal{Y}}_t\right)$ | −1.53 | 1.98 | | | | | 1.00 | 0.46 |
| (6) GDP growth $\log\left(\mathcal{Y}_t/\mathcal{Y}_{t-4}\right)$ | 2.00 | 1.51 | | | | | | 1.00 |

**Table 5:** Time-series and business cycle statistics for the belief wedges. *Panel A*: Belief wedge relative to a VAR forecast, time period 1982Q1–2019Q4. *Panel B*: Belief wedge relative to a VAR forecast, time period 1982Q1–2019Q4, excluding Great Recession. *Panel C*: Belief wedge relative to a VAR forecast, time period 1960Q3–2019Q4 (median inflation forecast not available for this period). *Panel D*: Belief wedge relative to the SPF forecast, time period 1982Q1–2019Q4.

| | | mean | std | correlation with output gap | GDP growth |
|---|---|---|---|---|---|
| Benchmark VAR model | Unemployment wedge | 0.52 | 0.57 | −0.49 | −0.28 |
| | Inflation wedge | 1.22 | 0.97 | −0.30 | −0.49 |
| | Principal component | 0.00 | 1.00 | −0.48 | −0.47 |
| CTW VAR model | Unemployment wedge | 0.57 | 0.54 | −0.52 | −0.24 |
| | Inflation wedge | 1.15 | 1.01 | −0.26 | −0.41 |
| | Principal component | 0.00 | 1.00 | −0.50 | −0.42 |
| DSSW VAR model | Unemployment wedge | 0.56 | 0.56 | −0.48 | −0.34 |
| | Inflation wedge | 1.23 | 0.94 | −0.33 | −0.38 |
| | Principal component | 0.00 | 1.00 | −0.50 | −0.45 |

**Table 6:** Belief wedges for alternative VAR specifications of the rational forecast. CTW: Christiano et al. (2011), DSSW: Del Negro et al. (2007).

(2016). DSSW use a smaller set of variables with a longer lag length of four periods: GDP growth, growth of consumption of nondurables and services, investment, wage growth, logarithm of hours worked per capita, GDP deflator, and the Federal Funds rate. All nominal quantities are per capita and deflated by the GDP deflator. CTW use two lags as in our benchmark specification but use a richer set of variables: relative price of investment goods, growth rate in real GDP per hour worked, GDP deflator, unemployment rate, capital utilization, real GDP per hour relative to real wage, consumption of nondurables and services relative to GDP, investment relative to GDP, job separation rate, job finding rate, vacancy rate, hours per person in labor force, and the Federal Funds rate. In both cases, we make sure include CPI inflation and unemployment rate as well to form the respective forecasts. The estimation period is 1960Q1–2019Q4.

The results are presented in Table 6 and Figure 17. The characteristics of the belief wedges as well as the resulting principal components are extremely similar across the specifications of the underlying VAR. In all cases, the wedges are countercyclical and track each other closely in the observed sample. The correlation between the principal components constructed using the benchmark model and those constructed using the CTW and DSSW VARs is 82% and 85%, respectively.

### D.5 Further cross-sectional evidence on the belief wedges

In this section, we provide further evidence on the cross-sectional relationship between household-level survey answer biases for alternative questions, documented in the Michigan Survey and the SCE.

In the cross-sectional analysis (except for Table 7), we do not convert unemployment responses using the procedure described in Appendix D.1, but encode categorical household-level responses on the forecasted change in the unemployment rate {down, same (or don't know), up} for household $i$ in demographic group $g$ and month $t$ as $\widetilde{u}_{i,g,t} \in \{-1, 0, 1\}$. We drop respondents aged 65 and above and those with missing responses. Population and group-level averages $\widetilde{u}_t$ and $\widetilde{u}_{g,t}$ then represent the share of respondents who forecast an increase in unemployment minus the share that forecasts a decrease. For the inflation responses, we drop households who indicate "don't know," have a missing response, or have extreme forecasts (above 20% or below −10%). The results are robust to keeping the extreme forecasts.

Table 7 reports the conditional time-series averages of the households' forecasts for different demographic groups in the Michigan Survey, displayed in Figure 4. More educated respondents and respondents with higher incomes overpredict inflation and unemployment less on average, but all demographic groups still overpredict both quantities. Moreover, demographic groups that on average overpredict inflation relatively

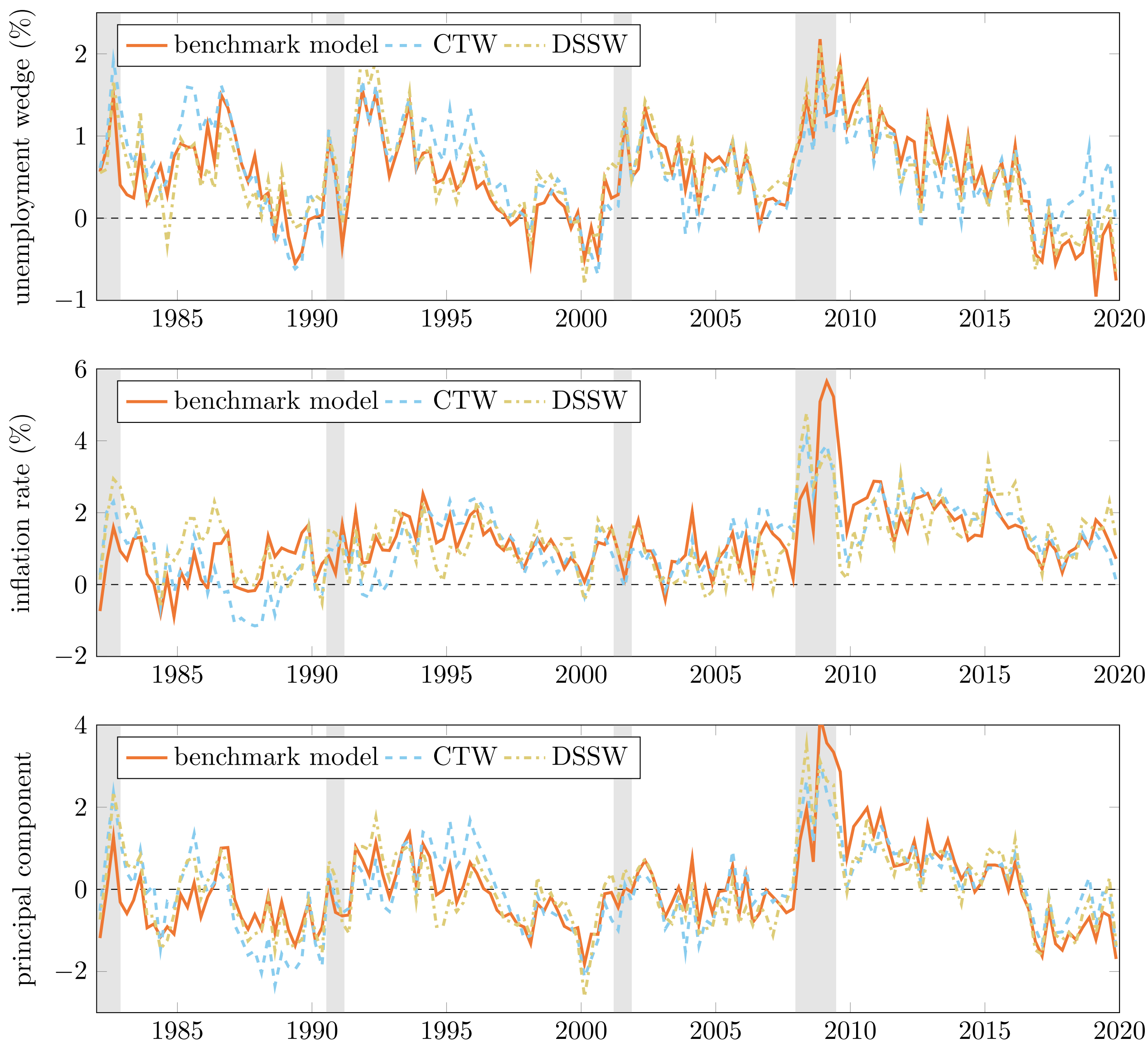


**Figure 17:** Belief wedges for alternative VAR specifications of the rational forecast. CTW: Christiano et al. (2011), DSSW: Del Negro et al. (2007). NBER recessions are shaded.

more also overpredict unemployment relatively more.

### D.5.1 Cross-sectional regressions

Tables 8–10 provide further details at the level of demographic groups and individual households. First, we ask whether in times when demographic group $g$ on average overpredicts inflation more relative to population, the group also overpredicts unemployment more relative to population. Table 8 summarizes the regression coefficients in time-series regressions of the form

$$\widetilde{u}_{g,t} - \widetilde{u}_t = \alpha_g + \beta_g \left[\widetilde{\pi}_{g,t} - \widetilde{\pi}_t\right] + \varepsilon_{g,t}, \tag{61}$$

| | actual | SPF | all | 18-34 | 35-44 | 45-54 | 55-64 | W | NC | NE | S |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi$ | 2.71 | 2.86 | 3.87 | 3.93 | 3.85 | 3.83 | 3.75 | 3.81 | 3.82 | 3.83 | 3.99 |
| $u$ | 6.14 | 6.05 | 6.55 | 6.44 | 6.58 | 6.63 | 6.63 | 6.54 | 6.55 | 6.61 | 6.53 |
| $u$ share | — | — | 14.8 | 11.3 | 15.6 | 17.2 | 17.1 | 14.3 | 15.1 | 16.6 | 14.0 |
| | | male | female | bottom | 2nd Q | 3rd Q | top | HS | SC | COL | GS |
| $\pi$ | | 3.33 | 4.36 | 4.78 | 4.10 | 3.59 | 3.12 | 4.38 | 3.84 | 3.41 | 3.30 |
| $u$ | | 6.45 | 6.64 | 6.71 | 6.58 | 6.52 | 6.42 | 6.63 | 6.55 | 6.46 | 6.48 |
| $u$ share | | 11.0 | 18.3 | 19.8 | 15.8 | 13.6 | 9.8 | 17.5 | 14.9 | 11.5 | 11.9 |

**Table 7:** Demographic characteristics of households' expectations on inflation ($\pi$) and the unemployment rate. The row labeled "$u$ share" is the percentage share of responses that the unemployment rate will increase minus the percentage share stating that the unemployment rate will decrease. The line labeled "$u$" is the average fitted unemployment rate forecast computed as in Appendix D.1. Time-series averages, all values are annualized and in percentages, time period 1982Q1–2019Q4. *Actual*: actual average inflation and unemployment rate; *SPF*: average SPF forecast; *all*: average household forecast; *18-34 etc*: age groups; *W*: West region; *NC*: North-Central; *NE*: North-East; *S*: South; *bottom, 2nd Q, 3rd Q, top*: income quartiles; *HS*: high school education; *SC*: some college; *COL*: college degree; *GS*: graduate studies.

where $\widetilde{u}_{g,t}$, $\widetilde{\pi}_{g,t}$ are the average forecasts of demographic group $g$ in month $t$, and $\widetilde{u}_t$, $\widetilde{\pi}_t$ are the average forecasts in month $t$ for the whole population. The estimated regression coefficients $\widehat{\beta}_g$ are all positive, and most of them are highly statistically significant.

Next, we investigate whether in times when *individual households* $i$ overpredict inflation more relative to the population, they also overpredict unemployment relatively more. The regression on the pooled sample of $N = 180,729$ household-level observations with demographic controls is

$$\widetilde{u}_{i,g,t} - \widetilde{u}_t = \alpha + \beta \left[\widetilde{\pi}_{i,g,t} - \widetilde{\pi}_t\right] + \delta' D_{i,g,t} + \varepsilon_{i,g,t},$$

where $\widetilde{u}_{i,g,t}$, $\widetilde{\pi}_{i,g,t}$ are the forecasts of household $i$ belonging to demographic group $g$ in month $t$ and $D_{i,g,t}$ is the vector of demographic group dummies. The estimated slope coefficient is $\widehat{\beta} = 2.26$ with a standard error of 0.04. We also run pooled regressions using differences between individual household forecasts and the group-specific average in the given month:

$$\widetilde{u}_{i,g,t} - \widetilde{u}_{g,t} = \alpha_c + \beta_c \left[\widetilde{\pi}_{i,g,t} - \widetilde{\pi}_{g,t}\right] + \varepsilon_{i,g,t} \tag{62}$$

for different demographic categorizations $c \in \{\text{pooled population, education, income, region, age, sex}\}$. Table 9 reports the estimates of regression coefficients $\widehat{\beta}_c$.

To show that these cross-sectional relationships are stable over time, we run the regressions month by month and for each demographic sorting $c$:

$$\widetilde{u}_{i,g,t} - \widetilde{u}_{g,t} = \alpha_{c,t} + \beta_{c,t} \left[\widetilde{\pi}_{i,g,t} - \widetilde{\pi}_{g,t}\right] + \varepsilon_{i,g,t}. \tag{63}$$

Table 10 shows the mean and standard deviation of the *distribution* of estimated coefficients $\widehat{\beta}_{c,t}$ for each of the categorizations. Regardless of the demographic categorization, around 96% of all the estimated coefficients $\widehat{\beta}_{c,t}$ are positive, and about 69% of them have a $t$-statistic larger than 1.96. Figure 18 plots the smoothed time series of the coefficients for the pooled population case and documents that the significantly positive cross-sectional relationship between the belief wedges is not specific to a particular subperiod in the data.

| | | | 18-34 | 35-44 | 45-54 | 55-65 | W | NC | NE | S |
|---|---|---|---|---|---|---|---|---|---|---|
| $100 \times \widehat{\beta}_g$ | | | 3.01 | 2.40 | 2.24 | 2.58 | 3.29 | 2.10 | 2.62 | 4.68 |
| std. err. | | | 0.99 | 0.79 | 0.81 | 0.75 | 0.90 | 0.88 | 0.84 | 0.86 |
| | male | female | bottom | 2nd Q | 3rd Q | top | HS | SC | COL | GS |
| $100 \times \widehat{\beta}_g$ | 3.84 | 5.53 | 0.66 | 0.93 | 3.28 | 1.53 | 5.76 | 5.46 | 3.11 | 6.38 |
| $100\times$ std. err. | 1.08 | 1.16 | 0.81 | 0.82 | 0.85 | 1.06 | 0.94 | 0.81 | 0.90 | 1.05 |

**Table 8:** Regression coefficients in regression (61) run separately for alternative demographic groups $g$, listed in the caption of Table 7. $100 \times \widehat{\beta}_g$ scales the left-hand side in the regression to percentage shares. Each regression involves $N = 456$ monthly observations.

| | population | age | region | sex | income | education |
|---|---|---|---|---|---|---|
| $100 \times \widehat{\beta}_c$ | 2.38 | 2.39 | 2.38 | 2.30 | 2.32 | 2.34 |
| $100\times$ std. err. | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| $N$ | 180,729 | 180,729 | 180,691 | 180,702 | 172,354 | 179,726 |

**Table 9:** Regression coefficients in pooled regression (62) for alternative demographic categorizations $c$. $100 \times \widehat{\beta}_c$ scales the left-hand side in the regression to percentage shares.

| | population | age | region | sex | income | education |
|---|---|---|---|---|---|---|
| average $100 \times \widehat{\beta}_{c,t}$ | 2.56 | 2.57 | 2.56 | 2.49 | 2.50 | 2.51 |
| std. dev. $100 \times \widehat{\beta}_{c,t}$ | 1.55 | 1.54 | 1.53 | 1.53 | 1.52 | 1.52 |
| months | 456 | 456 | 456 | 456 | 456 | 456 |
| $\# \ t > 0$ | 440 | 440 | 442 | 437 | 444 | 441 |
| $\# \ t > 1.96$ | 314 | 322 | 317 | 308 | 308 | 313 |

**Table 10:** Regression coefficients in regression (63) for alternative demographic categorizations $c$. "Months" indicates the number of monthly regressions we run in each case, and $\# \ t > 0$ and $\# \ t > 1.96$ indicate the number of regressions from that sample in which the estimate $\widehat{\beta}_{c,t}$ has a $t$-statistic larger than zero or 1.96, respectively. $100 \times \widehat{\beta}_c$ scales the left-hand side in the regression to percentage shares.

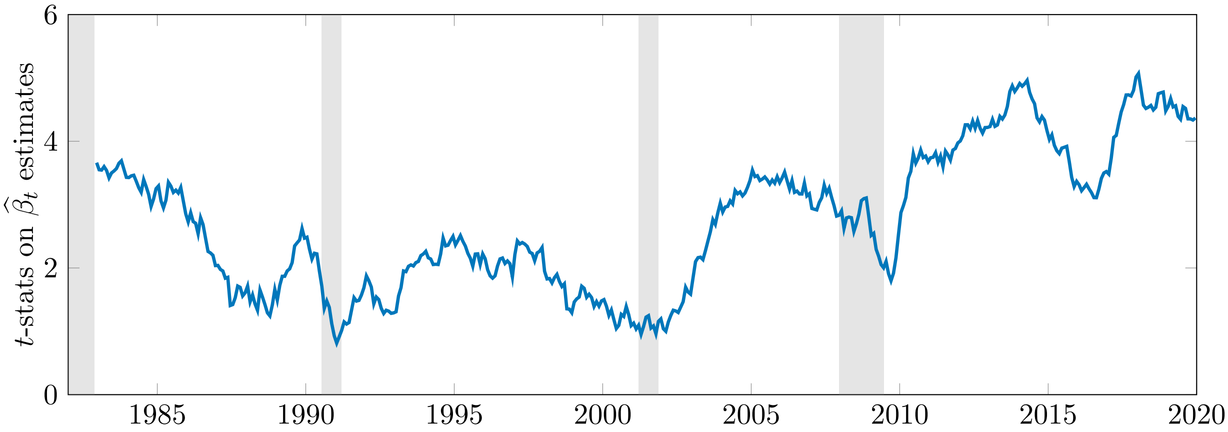


**Figure 18:** Smoothed (12-month moving average) $t$-statistics on the estimates $\widehat{\beta}_t$ in regression (63) for the case when the demographic sorting $c$ corresponds to the pooled population (i.e., $\widetilde{u}_{g,t} = \widetilde{u}_t$.)

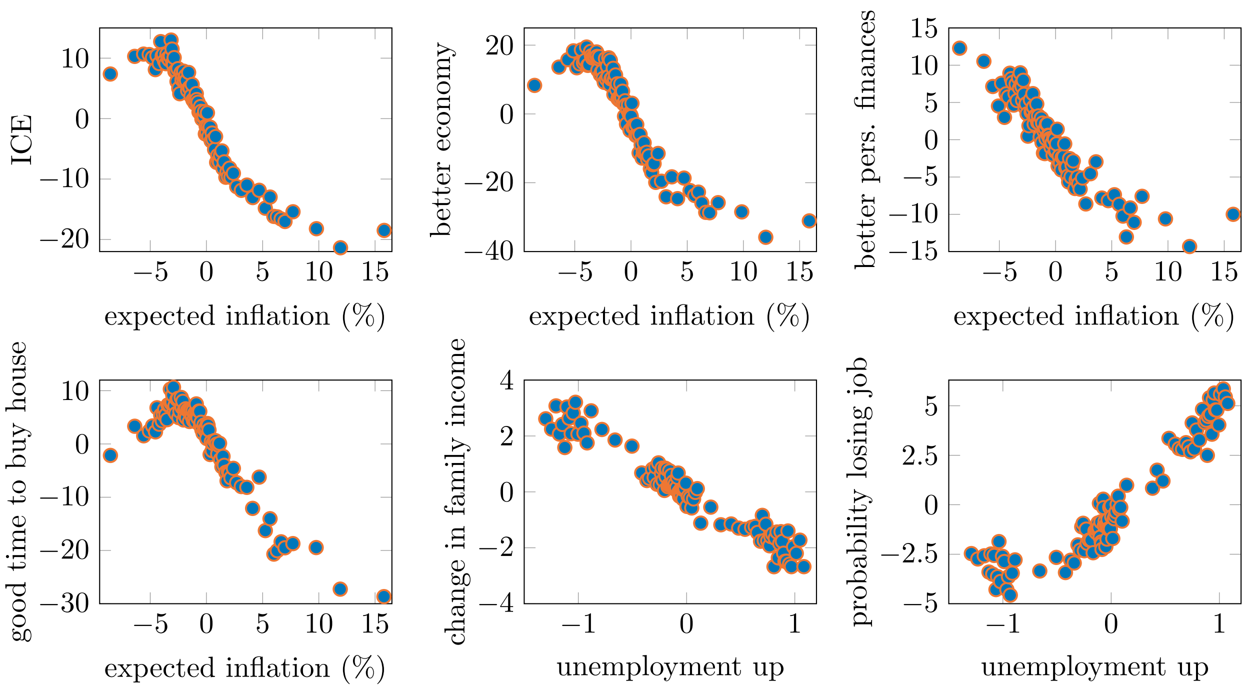


**Figure 19:** Bin scatter plots for household-level data in the Michigan Survey. Households sorted into percentile bins, with month fixed effects removed. One percent tails of the variable on the horizontal axis truncated. Axes labels described in the text.

### D.5.2 Scatter plots of household-level data from Michigan Survey and SCE

The tight cross-sectional relationship between the inflation and unemployment forecasts extends to forecasts of other aggregate and household-level variables. Figure 19 depicts this evidence in the form of bin scatter plots, paralleling the left panel of Figure 4 from the main text. The evidence is not exhaustive and holds for responses to other survey questions as well. In each panel, we remove month-specific means from the household-level forecasts of both variables, sort the forecasts on the variable depicted on the horizontal axis, and group them into percentile bins.

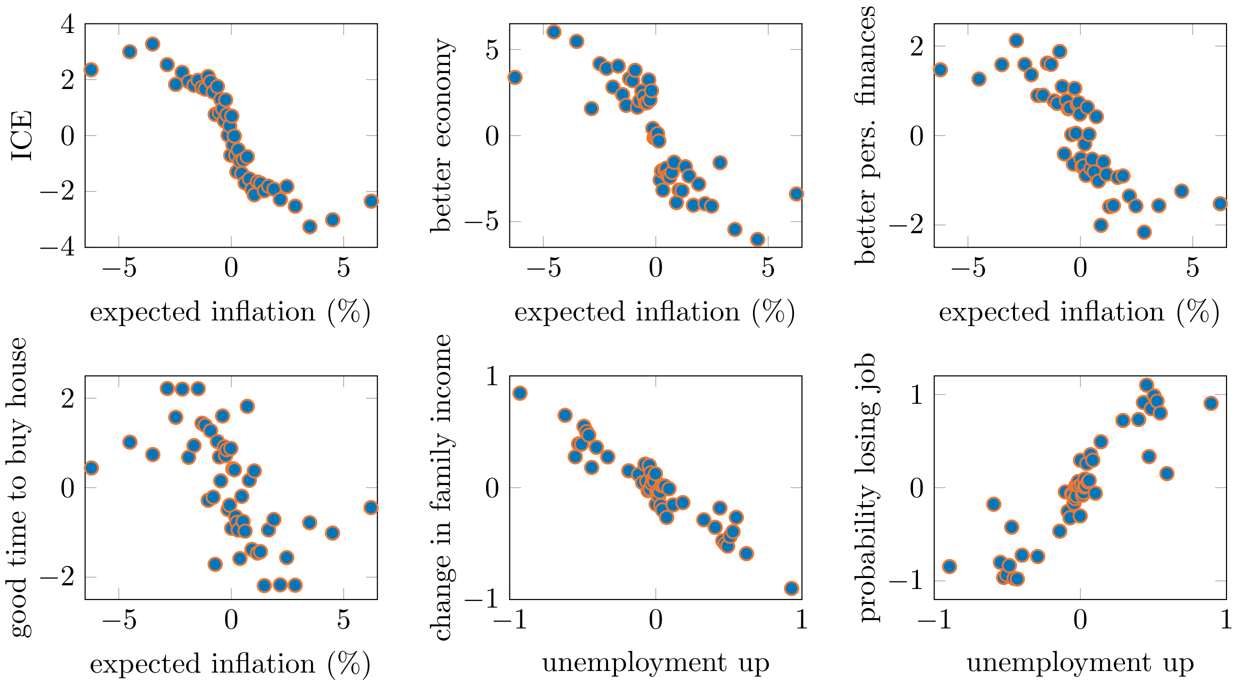


**Figure 20:** Bin scatter plots for the *panel component* of the household-level data in the Michigan Survey. Households sorted into two-percentile bins, with month and household fixed effects removed. One percent tails of the variable on the horizontal axis truncated. Axes labels described in the text.

Figure 20 replicates these bin scatter plots utilizing the panel component of the Michigan Survey. In the survey, a subsample of interviewed households is re-interviewed again six months after the initial interview. For this figure, we restrict our sample to this panel component, and remove month and household-level fixed effects.

The figures show that households strongly associate high inflation with bad times, both in terms of aggregate as well as household-specific quantities. Concerns about higher aggregate unemployment also translate into concerns about more adverse individual outcomes, household members correctly connect higher unemployment forecasts to higher chances of losing their own job, and to lower increases in individual incomes. The graphs for the panel component documents that this is not only due to individual fixed effects—increases in inflation forecasts are associated with more pessimistic updates of forecasts of macroeconomic quantities.

The specific variables depicted in the figures correspond to survey responses to the following questions:

- *expected inflation (%)*: 'By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?'
- *unemployment up*: 'How about people out of work during the coming 12 months—do you think that there will be more unemployment than now, about the same, or less?' Net share of 'more' minus 'less' responses.
- *ICE*: Index of Consumer Expectations, composite index computed from responses to a range of questions about future economic conditions.
- *better economy*: 'Now turning to business conditions in the country as a whole—do you think that during the next 12 months we'll have good times financially, or bad times, or what?' Net share of

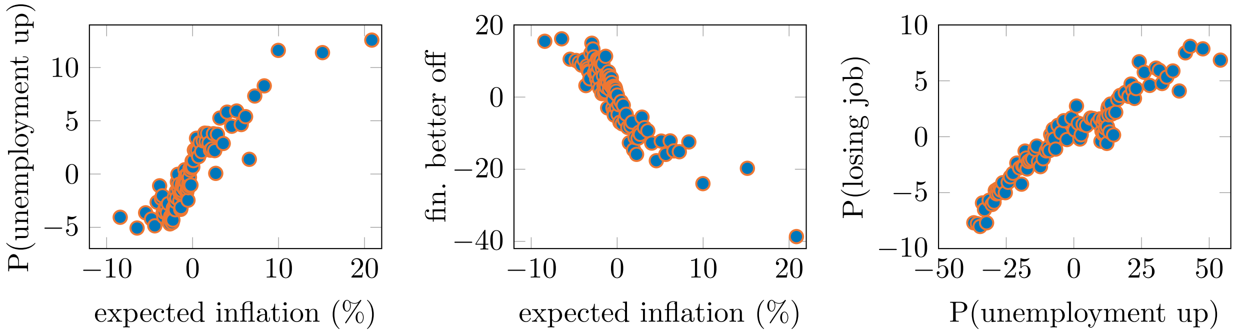


**Figure 21:** Bin scatter plots for household-level data in the Federal Reserve Bank of New York SCE. Households sorted into percentile bins, with month fixed effects removed. One percent tails of the variable on the horizontal axis truncated. Axes labels described in the text.

'good times' minus 'bad times' responses.

- *better pers. finances*: 'Now looking ahead—do you think that a year from now you will be better off financially, or worse off, or just about the same as now?' Net share of 'better off' minus 'worse off' responses.
- *good time to buy house*: 'Generally speaking, do you think now is a good time or a bad time to buy a house?' Net share of 'good time' minus 'bad time' responses.
- *change in family income*: 'By about what percent do you expect your income to (increase/decrease) during the next 12 months?'
- *probability losing job*: 'During the next 5 years, what do you think the chances are that you (or your husband/wife) will lose a job you wanted to keep?' Probability in percent.

The same pattern appears in the SCE administered by the Federal Reserve Bank of New York. Figure 21 shows that in the cross section, households that predict high inflation also assign a higher probability that unemployment will increase, and they expect they will be financially worse off. The last panel verifies that households consistently associated higher probabilities of aggregate unemployment with higher probabilities of losing their own job.

The SCE is a rotating panel that interviews the households monthly during the period of 12 months. Figure 22 replicates the analysis utilizing this panel component of the survey by removing household fixed effects as well. The graphs confirm that when households change their forecasts, they do so consistently with our theory. Increases in inflation expectations are associated with increases in the probability of higher unemployment and more adverse expectations of future financial conditions, while increases in expectations of higher unemployment are consistently accompanied with increases in expectations of losing respondents' own job.

The specific variables depicted in the figures correspond to survey responses to the following questions:

- *expected inflation (%)*: Calculated mean from the solicited probability distribution for the rate of inflation over the next 12 months.
- *P(unemployment up)*: 'What do you think is the percent chance that 12 months from now the unemployment rate in the U.S. will be higher than it is now?'

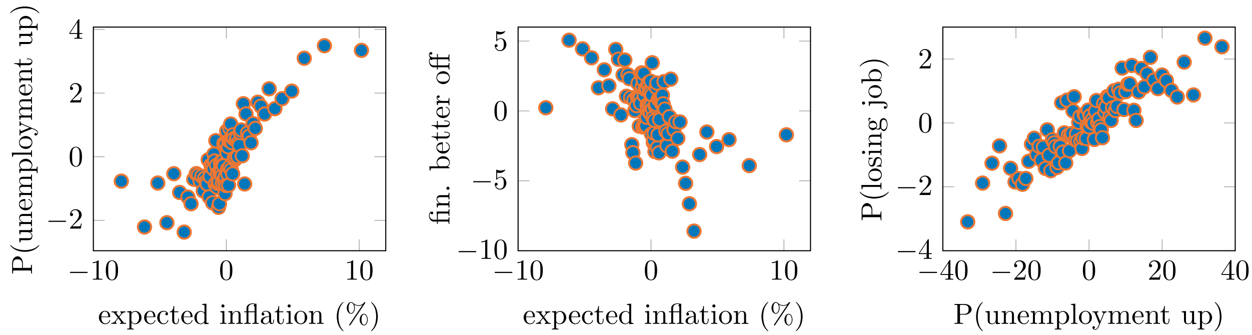


**Figure 22:** Bin scatter plots for the *panel component* of the household-level data in the Federal Reserve Bank of New York SCE. Households sorted into percentile bins, with month and household fixed effects removed. One percent tails of the variable on the horizontal axis truncated. Axes labels described in the text.

| | Data | | | | | Benchmark model | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Variable \ horizon** | 0 | 1 | 2 | 5 | 10 | 0 | 1 | 2 | 5 | 10 |
| inflation wedge | 1.00 | 0.68 | 0.54 | 0.34 | 0.25 | 1.00 | 0.71 | 0.51 | 0.18 | 0.04 |
| unemployment wedge | 1.00 | 0.67 | 0.64 | 0.41 | 0.06 | 1.00 | 0.71 | 0.51 | 0.18 | 0.04 |
| inflation | 1.00 | 0.81 | 0.63 | 0.32 | 0.20 | 1.00 | 0.93 | 0.77 | 0.23 | −0.04 |
| output | 1.00 | 0.96 | 0.89 | 0.59 | 0.16 | 1.00 | 0.84 | 0.69 | 0.31 | 0.01 |
| unemployment | 1.00 | 0.98 | 0.95 | 0.76 | 0.33 | 1.00 | 0.65 | 0.47 | 0.07 | −0.13 |

**Table 11:** Autocorrelation functions for macroeconomic quantities and belief wedges. The sample period for the data is 1982Q1–2019Q4. Output is detrended, inflation rate is the 4-quarter change in the price index.

- *fin. better off*: 'Looking ahead, do you think you (and any family living with you) will be financially better or worse off 12 months from now than you are these days?' Net share of 'better' minus 'worse' responses.
- *P(losing job)*: 'What do you think is the percent chance that you will lose your current (main) job during the next 12 months?'

# E Model fit

In this section, we provide further evidence on the fit of the benchmark model with the data. We first report additional unconditional moments obtained from the model dynamics that accompany those reported in Table 2, and then extend evidence from Section 6.1 on impulse responses of belief wedge components constructed using local projections.

## E.1 Unconditional moments for the benchmark model

Tables 11 to 13 provide a comparison of the theoretical moments implied by the model with the data. Table 11 shows that the model produces the right amount of persistence of the belief wedges, and a somewhat higher persistence of inflation. The model does not have enough persistence to produce the particularly high autocorrelation of unemployment; this is closely related to the absence of a hump-shaped response of unemployment to the belief shock discussed in Section 6.1.

| Variable | Data (1) | (2) | (3) | (4) | (5) | Benchmark model (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) inflation wedge | 1.00 | 0.34 | −0.56 | −0.30 | 0.26 | 1.00 | 1.00 | 0.33 | −0.67 | 0.86 |
| (2) unemployment wedge | | 1.00 | −0.15 | −0.49 | 0.61 | | 1.00 | 0.33 | −0.67 | 0.86 |
| (3) inflation | | | 1.00 | 0.10 | 0.03 | | | 1.00 | −0.82 | 0.39 |
| (4) output | | | | 1.00 | −0.87 | | | | 1.00 | −0.74 |
| (5) unemployment | | | | | 1.00 | | | | | 1.00 |

**Table 12:** Unconditional correlation matrix for macroeconomic quantities and belief wedges. The sample period for the data is 1982Q1–2019Q4. Output is detrended, inflation rate is the 4-quarter change in the price index.

| Shock \ horizon | inflation 1 | 10 | 100 | output 1 | 10 | 100 | unemployment 1 | 10 | 100 |
|---|---|---|---|---|---|---|---|---|---|
| belief shock $w_t^\theta$ | 0.40 | 0.31 | 0.33 | 0.78 | 0.51 | 0.51 | 0.83 | 0.84 | 0.84 |
| technology shock $w_t^a$ | 0.58 | 0.66 | 0.64 | 0.14 | 0.45 | 0.45 | 0.08 | 0.08 | 0.08 |
| monetary policy shock $w_t^r$ | 0.02 | 0.03 | 0.03 | 0.08 | 0.04 | 0.04 | 0.09 | 0.08 | 0.08 |

**Table 13:** Theoretical variance decomposition of forecast errors for inflation, output, and unemployment rate. Inflation rate is the 4-quarter change in the price index.

In Table 12, we display the unconditional covariance matrix that expands the results from Table 2. As mentioned in Section 4.3, the model reproduces the countercyclical comovement of wedges with output and unemployment but misses the unconditional dynamics of inflation—in the data sample, inflation is essentially acyclical, while the model produces countercyclical inflation. In order to understand the underlying source of this unconditional correlation, Table 13 displays the variance decomposition of forecast errors for alternative macroeconomic variables. The model attributes a substantial share of variation in inflation to technology shocks, while the belief shock predominantly drives output and unemployment. This is very similar to the variance decomposition between ambiguity and TFP shocks at business cycle frequencies produced in Ilut and Schneider (2014), who allocate most of the variation in inflation at business cycle frequencies to the TFP shock, while real variables are driven by the ambiguity shock, which drives the subjective conditional mean of TFP growth.

## E.2 Impulse responses constructed using local projections

In Section 6.1, we constructed impulse responses to an innovation in the belief shock $\theta_t$ using local projections proposed in Jordà (2005). As shown in Figure 12 in the main text, a positive innovation to the belief wedge (a positive shock $w_t^\theta$), representing an increase in pessimism, predicts a substantial increase in the unemployment rate and an initial brief increase followed by a modest but persistent decrease in the inflation rate. Unemployment and inflation wedges increase, reflecting the increase in the degree of pessimism.

Figure 23 provides additional evidence on the responses of the inflation and unemployment belief wedges by decomposing the wedge responses into the separate contributions of the Michigan and SPF forecasts.

The top row shows that just like in the model, the increase in the inflation belief wedge after a positive belief shock uncovered by the local projection is driven by an increase in the Michigan forecast, accompanied by a mildly negative response of the SPF forecast. Combining these responses yields the responses of the inflation wedge in Figure 12.

The bottom row of the figure presents the predicted responses of the unemployment wedge components, given by the evolution over time of the one-year ahead forecasts of the *change* in the unemployment rate

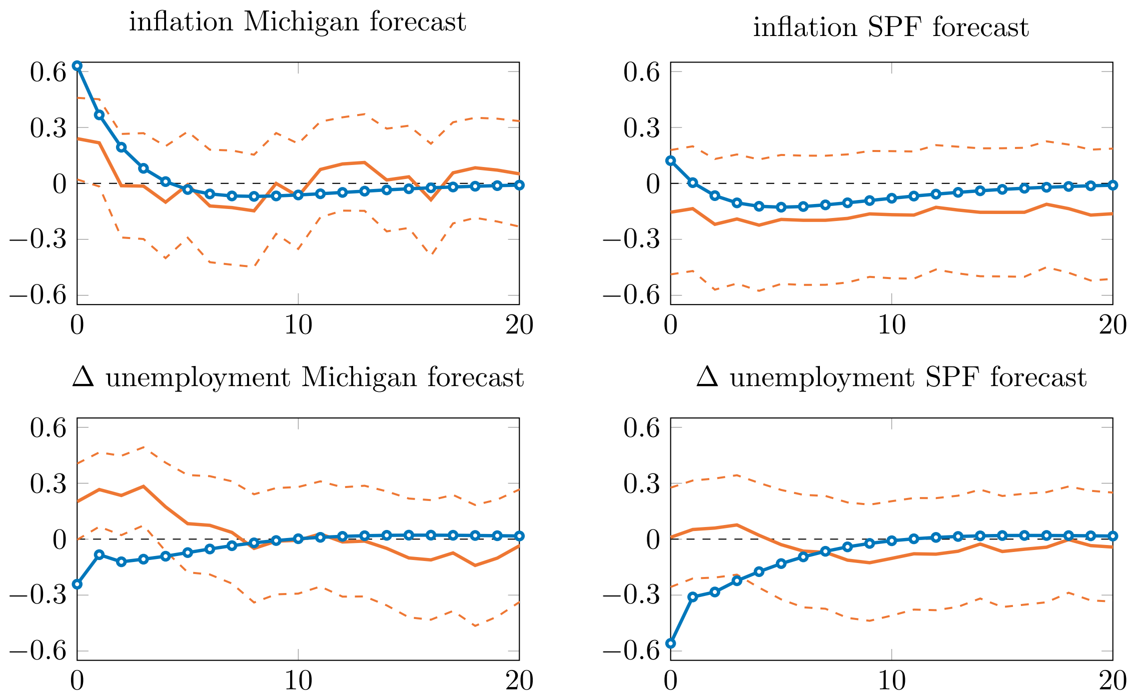


**Figure 23:** Local projection coefficients to the innovation in the principal component of the belief wedges (solid orange lines), constructed as the difference between the Michigan and SPF forecasts. Responses are in percentage points, inflation rate is annualized. Δ unemployment forecast displays the projection of the forecast of the change in the unemployment rate. Dashed lines represent ±1 standard deviation bands constructed using heteroskedasticity-robust standard errors. Model-implied impulse responses are displayed using blue lines with circles. Horizontal axis is in quarters.

in response to an initial shock to $w_t^\theta$. After the positive belief shock, households in the Michigan survey start forecasting further increases in the unemployment rate for about 6–8 quarters, while the SPF forecast is muted and somewhat negative after 2–3 years. Combining these temporal patterns generates the large and persistent increase in the unemployment wedge. The difference compared to the model stems from the absence of the hump-shaped unemployment response in the model—in the model, the unemployment rate increases on impact of the belief shock and then starts declining, which is rationally reflected in the response of the SPF forecast, while pessimistic households expect a much more gradual decline in the unemployment rate after the initial increase. However, the difference between the two responses is the same in the data as in the model, which generates a consistent responses of the unemployment wedge in Figure 12.

The key takeaway from the figure is the observation that empirically, the belief shocks that generate fluctuations in the wedges are primarily driven by movements in the Michigan forecasts, emphasizing the role of belief fluctuations in the household survey.

Finally, in Figure 24 we document consistent behavior of the belief wedges constructed using the SPF forecasts and VAR forecasts as rational forecasts. In particular, we use local projections to construct impulse responses of the VAR wedges to an innovation of the principal component constructed from the SPF wedges, as in Figures 12 and 23. The left column shows that VAR wedges also increase in response to the positive belief shock. The right column depicts that the VAR forecasts respond in a similar way as the SPF forecasts to the belief shock, with a somewhat more pronounced negative response of the inflation VAR forecast.

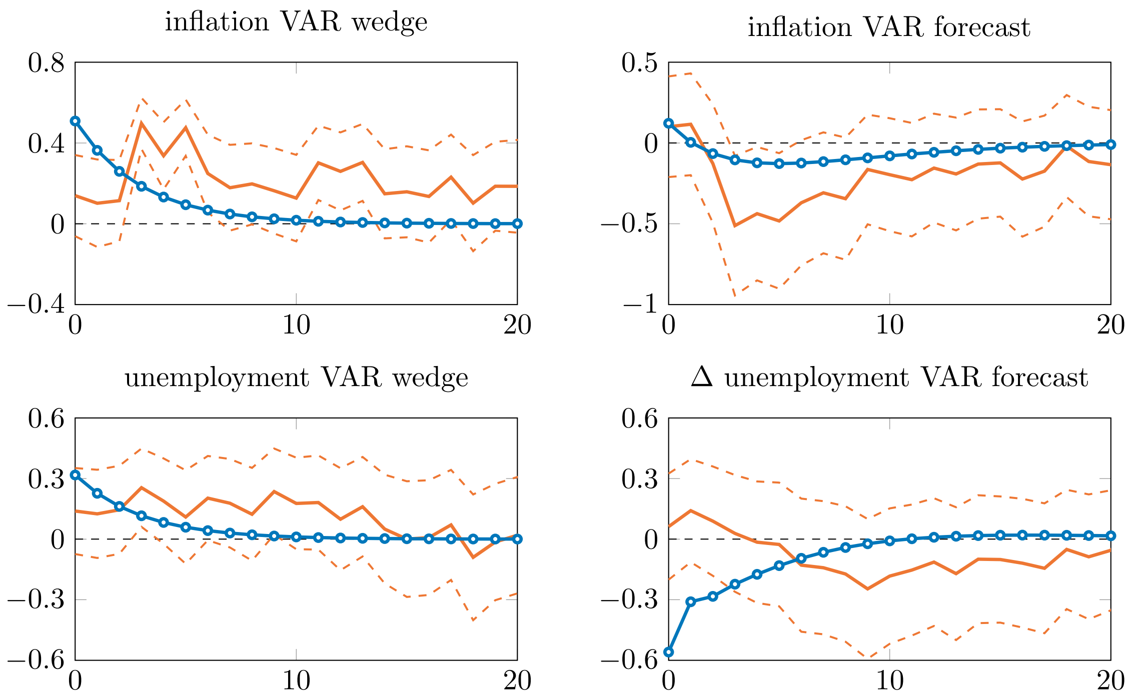


**Figure 24:** Local projection coefficients to the innovation in the principal component of the belief wedges (solid orange lines), constructed as the difference between the Michigan and SPF forecasts. Responses are in percentage points, inflation rate is annualized. $\Delta$ unemployment forecast displays the projection of the forecast of the change in the unemployment rate. Dashed lines represent $\pm 1$ standard deviation bands constructed using heteroskedasticity-robust standard errors. Model-implied impulse responses are displayed using blue lines with circles. Horizontal axis is in quarters.

# F Robustness checks

## F.1 Construction of the belief shock

In Section 4.2, we used the first principal component of the unemployment and inflation wedges as the observable counterpart of the belief shock. As an alternative, we estimate the belief shock using the following hidden factor model

$$\begin{aligned} \theta_t - \overline{\theta} &= \rho_\theta \left(\theta_{t-1} - \overline{\theta}\right) + v_t \\ y_t &= H\theta_t + \varepsilon_t, \end{aligned}$$

where $y_t = \left(\overline{\Delta}_t^{(4)}(\pi), \overline{\Delta}_t^{(4)}(u)\right)'$ are the survey wedges, $H$ is a $2 \times 1$ vector of factor loadings, and $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ are the measurement errors in the survey data with $\varepsilon_{i,t} \sim N\left(0, \sigma_i^2\right)$. We estimate the model using Bayesian methods (Chib and Greenberg (1996)), imposing a relatively flat prior with $\rho_\theta \sim N(0.5, 0.25)$, $\sigma_i^2 \sim IG(2,1)$, and $H_i \mid \sigma_i \sim N\left(0, \sigma_i^2\right)$. In Figure 25, we compare the mean path under the posterior estimate with the principal component. The correlation between the two time series is 0.87.

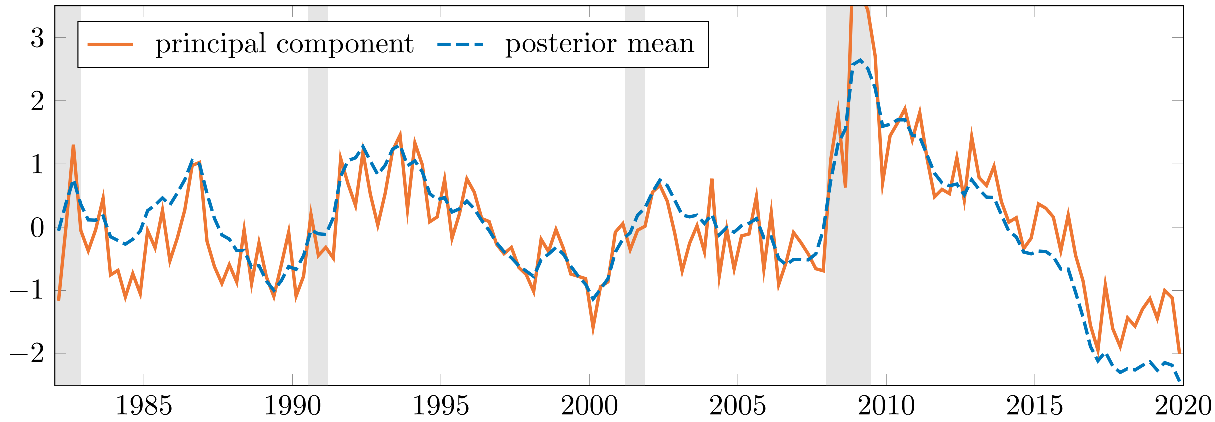


**Figure 25:** Comparison of the first principal component of the belief wedges (solid line) with the mean path under the posterior estimate of the hidden factor model (dashed line). Both series are standardized.

## F.2 Results for the model with fluctuations in pessimism induced by productivity shocks

In Section 5.3, we study the model in which fluctuations in the belief shock $\theta_t$ are induced by TFP innovations. Figures 26 and 27 plot the impulse response functions and simulated paths, respectively, for this specification. In order to make the economy comparable to the benchmark model, we choose a calibration that leads to the same properties of the unemployment and inflation wedges. (See column titled '$\theta(a_t)$' in Table 2.) This leads us to choose $\mu_\theta = 1.7$, $c_a = -156.25$, $\xi_p = 0.66$, and $\xi_w = 0.6$. All other parameters are as in the benchmark model reported in Table 1.

## F.3 Results for the model with flexible wages

The last column of Table 2 presents unconditional moments for the version of the model with flexible wages ($\lambda = 0$). In this version of the model, we keep the parameters of the belief shock $\theta_t$ unchanged. Figure 28 represents the corresponding impulse responses to the belief shock innovation using the red dashed lines, relative to the benchmark model plotted with blue solid lines. With flexible wages, the bargained wage decreases aggressively after an increase in pessimism, so that the decline in firm valuation is substantially less severe, firms continue to post vacancies, and the increase in unemployment is more moderate. This is further reinforced by endogenous determination of the subjective belief. Lower macroeconomic uncertainty results in a smaller covariance between the continuation value and future cash flows, so, for a given increase in $\theta_t$, the increase in the pessimistic bias is more modest, which further increases the incentives to hire.

The flexible wage model also produces a negligible reaction of inflation. The increase in pessimism is more modest relative to the benchmark model, hence the pessimistic price-setting firms do not subjectively expect such a high increase in production costs. Expectations of declining demand and increasing production costs offset each other, generating a steady aggregate price level.

As documented in Table 2, the flexible price model generates smaller and less volatile belief wedges than those in the data and the benchmark model. Since the mean and volatility of the $\theta_t$ are not structural parameters, we relibrate them to obtain comparable volatility of the unemployment belief wedge ($\mu_\theta = 8.5$, $\sigma_\theta = 4.9$). Impulse responses for the recalibrated flexible price model are depicted with green dash-dotted lines in Figure 28. The on-impact response of the unemployment rate and the unemployment belief are now similar to those in the benchmark model. However, the dynamics of inflation are substantially different now.

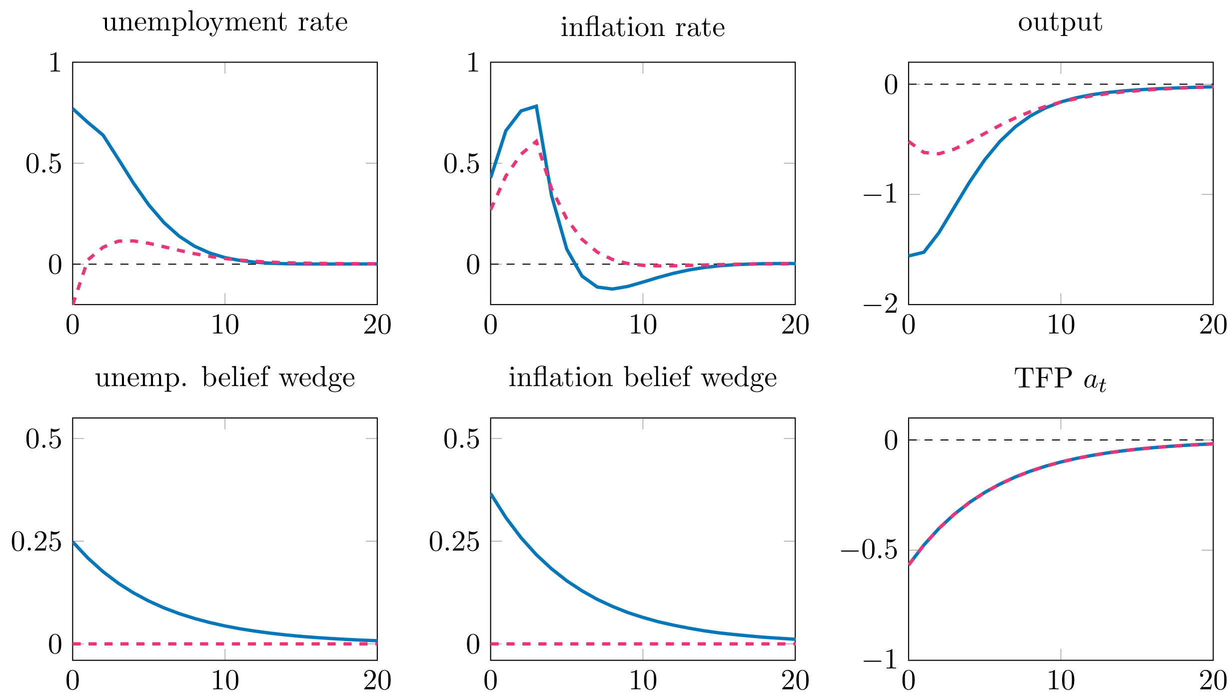


**Figure 26:** Impulse response functions to a *negative* TFP innovation $w^a$ under the data-generating measure $P$ in the model with fluctuations in $\theta_t$ induced by TFP innovations determined by equation (21) (solid line), and in the corresponding rational model (dashed line). Output response is in percentages, and unemployment rate, inflation rate, TFP, and belief wedges are in percentage points. Inflation rate is the 4-quarter change in the price index. Horizontal axis is in quarters.

Inflation falls on impact, due to a substantial increase in the concerns about the demand effects of the shock. These concerns dominate the subjective belief about the adverse cost effects of the TFP shocks, and, as a result, the inflation belief wedge turns negative on average (with a mean of $-0.23\%$), and falls (becomes more negative) on impact of the belief shock.

This counterfactual behavior of the inflation belief wedge reflects once again a key economic restriction imposed by the survey data. In order for the model to generate a positive and countercyclical inflation belief wedge, subjective concerns about supply-type shocks need to dominate agents' pessimistic beliefs. We therefore cannot arbitrarily increase the volatility of the belief shock $\sigma_\theta$ to match the volatility of macroeconomic quantities without running into the constraint imposed by the behavior of the belief wedges observed in the data.

# G Model with uninsurable idiosyncratic risk

In this appendix, we derive the belief distortion implied by the endowment economy model with uninsurable idiosyncratic risk outlined in Section 5.4 and motivated by Constantinides and Duffie (1996). Consumption of household $i$ is given by $C_t^i = \delta_t^i C_t$, with aggregate and idiosyncratic components specified in growth rates

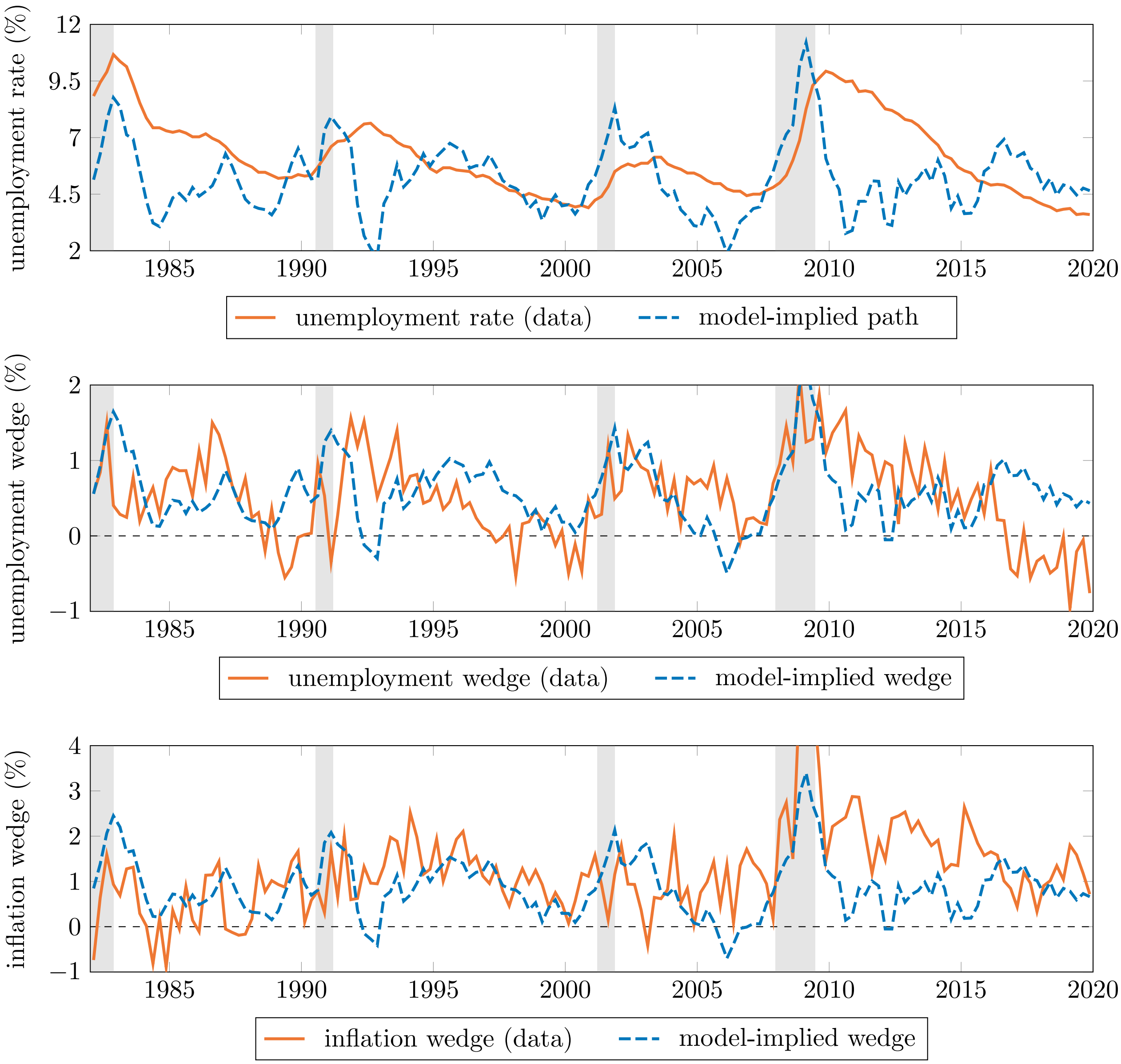


**Figure 27:** Data and model-implied time series for unemployment and belief wedges in the model with fluctuations in $\theta_t$ induced by TFP innovations determined by equation (21).

as

$$
\begin{aligned}
\Delta c_{t+1} &\doteq \log C_{t+1} - \log C_t = \bar{c} \\
\frac{\delta_{t+1}^i}{\delta_t^i} &= \exp\left(-\eta_{t+1}^i \sigma_{t+1} - \frac{1}{2}\sigma_{t+1}^2\right) \\
\sigma_{t+1}^2 &= (1-\psi_\sigma)\,\bar{\sigma}^2 + \psi_\sigma \sigma_t^2 + \sigma_t \psi_{\sigma w} w_{t+1}.
\end{aligned}
$$

with $\eta_{t+1}^i \sim N(0,1)$. Shocks $\eta_{t+1}^i$ are iid across households and over time, with $E\left[\eta_{t+1}^i\right] = 1$, which ensures that household consumption aggregates to $C_t$. Household preferences are logarithmic, $u(C) = (1-\beta)\log C$.

The household is endowed with subjective belief implied by preferences (2), with a constant penalty

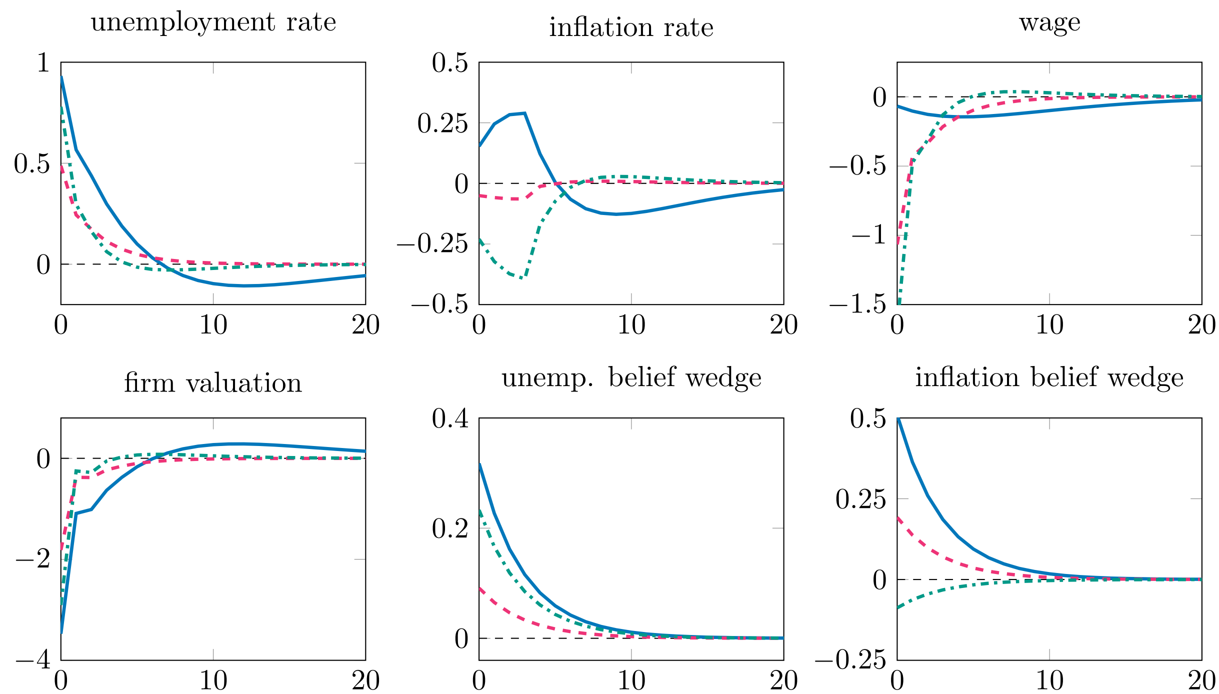


**Figure 28:** Impulse response functions to the belief shock innovation $w^{\theta}$ for the benchmark model (solid blue lines), model with flexible wages (dashed red lines), and model with flexible wages and recalibrated volatility of the belief shock $\theta_t$. Wage and firm value responses are in percentages, and unemployment rate, inflation rate, and belief wedges are in percentage points. Inflation rate is the 4-quarter change in the price index. Horizontal axis is in quarters.

parameter $\theta_t = \theta$. The continuation value recursion of the household satisfies

$$V_t^i = (1-\beta)\log C_t^i - \frac{\beta}{\theta}\log E_t\left[\exp\left(-\theta V_{t+1}^i\right)\right].$$

Applying results from Appendix B.7, we define $v_t^i = V_t^i - \log C_t^i$, implying the recursion

$$v_t^i = -\frac{\beta}{\theta}\log E_t\left[\exp\left(-\theta\left(v_{t+1}^i + \Delta c_{t+1}^i\right)\right)\right], \tag{64}$$

with individual consumption growth following

$$\Delta c_{t+1}^i = \log\frac{\delta_{t+1}^i}{\delta_t^i} + \Delta c_{t+1} = -\eta_{t+1}^i\sigma_{t+1} - \frac{1}{2}\sigma_{t+1}^2 + \bar{c}.$$

Denote $\widehat{E}_{t+1}[\cdot]$ the expectation operator that conditions on the information set consisting of time-$t$ information and time-$t+1$ aggregate variables. We conjecture the solution of the form $v_t^i = \bar{v}_\sigma\sigma_t^2 + \bar{v}_0$. The fact is that the scaled continuation value does not depend on the identity of the household is the consequence of the permanent-shock nature of idiosyncratic uncertainty. Substituting into the recursion (64) yields restrictions on the coefficients $\bar{v}_\sigma$ and $\bar{v}_0$. In particular, the restriction on $\bar{v}_\sigma$ is given in the form of a Riccati equation

$$\bar{v}_\sigma = \beta\left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right)\psi_\sigma - \frac{\beta\theta}{2}\left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right)^2\psi_{\sigma w}\psi_{\sigma w}' \doteq f(\bar{v}_\sigma).$$

The term on the right-hand side denoted as $f(\bar{v}_\sigma)$ concave in $\bar{v}_\sigma$, with $f(0)<0$ and $f'(0)>0$, which means that if real solutions exist, they will be negative, and the larger (smaller in magnitude) solution will be stable and can be obtained by backward iterations $\bar{v}_\sigma^{(n+1)} = f\left(\bar{v}_\sigma^{(n)}\right)$ from $\bar{v}_\sigma^{(0)} = 0$. Real roots will exist if and only if

$$(1-\beta\psi_\sigma)^2 - \beta\theta(1+\theta)\psi_{\sigma w}\psi_{\sigma w}' \geq 0.$$

The one-period belief distortion of household $i$ is given by

$$\begin{aligned} m_{t+1}^i &= \frac{\exp\left(-\theta V_{t+1}^i\right)}{E_t\left[\exp\left(-\theta V_{t+1}^i\right)\right]} = \frac{\exp\left(-\theta\left(v_{t+1}^i + \Delta c_{t+1}^i\right)\right)}{E_t\left[\exp\left(-\theta\left(v_{t+1}^i + \Delta c_{t+1}^i\right)\right)\right]} && (65)\\ &= \frac{\exp\left(-\theta\left(\left(\bar{v}_\sigma - \frac{1}{2}\right)\sigma_{t+1}^2 - \eta_{t+1}^i\sigma_{t+1}\right)\right)}{E_t\left[\exp\left(-\theta\left(\left(\bar{v}_\sigma - \frac{1}{2}\right)\sigma_{t+1}^2 - \eta_{t+1}^i\sigma_{t+1}\right)\right)\right]} && (66)\end{aligned}$$

To establish subjective belief wedges for aggregate variables, we can condition down $m_{t+1}^i$ to

$$\begin{aligned} \widehat{E}_{t+1}\left[m_{t+1}^i\right] &\doteq \widehat{m}_{t+1} = \frac{\exp\left(-\theta\left(\bar{v}_\sigma - \frac{1}{2}\right)\sigma_{t+1}^2\right)\widehat{E}_{t+1}\left[\exp\left(\theta\eta_{t+1}^i\sigma_{t+1}\right)\right]}{E_t\left[\exp\left(-\theta\left(\bar{v}_\sigma - \frac{1}{2}\right)\sigma_{t+1}^2\right)\widehat{E}_{t+1}\left[\exp\left(\theta\eta_{t+1}^i\sigma_{t+1}\right)\right]\right]}\\ &= \frac{\exp\left(-\theta\left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right)\sigma_t\psi_{\sigma w}w_{t+1}\right)}{E_t\left[\exp\left(-\theta\left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right)\sigma_t\psi_{\sigma w}w_{t+1}\right)\right]}.\end{aligned}$$

Since $\widehat{m}_{t+1}$ does not depend on the identity of the household, households share the same subjective belief with respect to aggregate variables. As in (18), consider an aggregate variable $z_t = \bar{z}'x_{1t}$ with $x_{1t}$ that follows (10). Then the belief wedge is given by

$$\begin{aligned} \Delta_t^{(1)}(z) &= \widetilde{E}_t\left[z_{t+1}\right] - E_t\left[z_{t+1}\right] = \bar{z}'\psi_w\widetilde{E}_t\left[w_{t+1}\right]\\ &= -\theta\sigma_t\bar{z}'\psi_w\psi_{\sigma w}'\left(\bar{v}_\sigma - \frac{1}{2}(1+\theta)\right)\end{aligned}$$

This is the counterpart of the belief wedge (18) from the homoskedastic model.

To show that a no-trade equilibrium exists under the specification of consumption processes conjectured above, consider a cash flow process $D_t$ contingent on aggregate risk with equilibrium price $P_t$. Optimal portfolio choice of household $i$ implies the Euler equation

$$P_t = \widetilde{E}_t^i\left[s_{t+1}^i\left(P_{t+1} + D_{t+1}\right)\right] = E_t\left[m_{t+1}^i s_{t+1}^i\left(P_{t+1} + D_{t+1}\right)\right]$$

where $\widetilde{E}_t^i[\cdot]$ is the expectation operator under the subjective belief of household $i$ and $s_{t+1}^i = \beta\left(C_{t+1}^i/C_t^i\right)^{-1}$ is the corresponding stochastic discount factor. Since the cash flow and price process are only contingent on aggregate state variables, we can condition down and write

$$P_t = E_t\left[\widehat{E}_{t+1}\left[m_{t+1}^i s_{t+1}^i\right]\left(P_{t+1} + D_{t+1}\right)\right].$$

Following the same derivation as in the construction of $\widehat{m}_{t+1}$, we obtain

$$\begin{aligned}
\widehat{E}_{t+1}\left[m^i_{t+1}s^i_{t+1}\right] &\doteq \widehat{s}_{t+1}=\beta\frac{\widehat{E}_{t+1}\left[\exp\left(-\theta v^i_{t+1}-(1+\theta)\,\Delta c^i_{t+1}\right)\right]}{E_t\left[\widehat{E}_{t+1}\left[\exp\left(-\theta v^i_{t+1}-\theta\Delta c^i_{t+1}\right)\right]\right]}\\
&= \beta\exp\left(-\bar{c}+(1+\theta)\left[(1-\psi_\sigma)\,\bar{\sigma}^2+\psi_\sigma\sigma_t^2+\left(-\theta\bar{v}_\sigma+\frac{1}{2}(1+\theta)\right)\sigma_t^2\psi_{\sigma w}\psi'_{\sigma w}\right]\right)\cdot\\
&\quad\cdot\frac{\exp\left(\left(-\theta\bar{v}_\sigma+\frac{1}{2}(1+\theta)(2+\theta)\right)\sigma_t\psi_{\sigma w}w_{t+1}\right)}{E_t\left[\exp\left(\left(-\theta\bar{v}_\sigma+\frac{1}{2}(1+\theta)(2+\theta)\right)\sigma_t\psi_{\sigma w}w_{t+1}\right)\right]}
\end{aligned}$$

The belief adjusted marginal rate of substitution conditioned down on aggregate variables is therefore identical across households at the prevailing consumption processes, which yields a no-trade equilibrium as in Constantinides and Duffie (1996). The time-varying nature of idiosyncratic risk induces both time-variation in risk-free discounting, manifested by the first line of the last expression, as well as in belief-adjusted prices of risk, captured by the last term.

## H Alternative models of belief updating

In this section, we provide a theoretical justification for regression (22) using two models of information processing. The first is a sticky information model in the spirit of Mankiw and Reis (2002). Assume that the forecasted variable $z_t$ follows an AR(1) process

$$z_t=\rho z_{t-1}+w_t$$

with iid innovations $w_t$ and $\rho\in[0,1]$. Under full information, the $j$-period-ahead forecast is $E_t[z_{t+j}]=\rho^j z_t$. Under sticky information, each agent updates her information about the current state with probability $1-\lambda\in(0,1]$. At every time $t$, a fraction $(1-\lambda)\,\lambda^k$ of agents last observed the state of the process at time $t-k$ (the case $\lambda=0$ thus corresponds to the full information model). The cross-sectional average of the individual forecasts at time $t$, which plays the role of the aggregate forecast in (22), is therefore given by

$$\widetilde{E}_t[z_{t+j}]=(1-\lambda)\sum_{k=0}^{\infty}\lambda^k E_{t-k}[z_{t+j}]=(1-\lambda)\sum_{k=0}^{\infty}\lambda^k\rho^{j+k}z_{t-k},$$

where $E_{t-k}[z_{t+j}]$ is the time-$t$ forecast of an agent who has last updated her information at time $t-k$. Since

$$z_{t-k}=\sum_{m=0}^{\infty}\rho^m w_{t-k-m},$$

we get

$$\widetilde{E}_t[z_{t+j}]=(1-\lambda)\sum_{k=0}^{\infty}\lambda^k\rho^{j+k}\sum_{m=0}^{\infty}\rho^m w_{t-k-m}=\sum_{k=0}^{\infty}\left(1-\lambda^{k+1}\right)\rho^{j+k}w_{t-k},$$

which can be represented recursively as

$$\widetilde{E}_t[z_{t+j}]=(1-\lambda)\,\rho^j z_t+\lambda\rho\widetilde{E}_{t-1}[z_{t-1+j}]\,.$$

This yields the expression for forecast errors of the average forecast

$$z_{t+j} - \widetilde{E}_t [z_{t+j}] = \lambda \rho^j z_t - \lambda \rho \widetilde{E}_{t-1} [z_{t-1+j}] + \sum_{m=0}^{j-1} \rho^m w_{t+j-m}.$$

This corresponds to regression (22) with $b_0 = 0$, $b_z = \lambda\rho^j \in [0, 1)$, and $b_f = -\lambda\rho \in (-1, 0]$. The regression coefficients reduce to $b_z = b_f = 0$ in the absence of information frictions ($\lambda = 0$).

The second model is a noisy information model motivated by Lucas (1972), Sims (2003), and Woodford (2003a). Specifically, $z_t$ follows again an AR(1) process but is not observable. Instead, each agent $i$ receives a combination of a public signal $y_t = z_t + \chi_t$, $\chi_t \sim N\left(0, \sigma_\chi^2\right)$ that is common for everybody and an idiosyncratic private signal $y_{it} = z_t + \eta_{it}$, $\eta_{it} \sim N\left(0, \sigma_\eta^2\right)$. The state space system can then be written as

$$\begin{aligned} z_t &= \rho z_{t-1} + w_t \qquad w_t \sim N(0, \Sigma_w) \\ s_{it} &= h z_t + v_{it} \qquad v_{it} \sim N(0, \Sigma_v) \end{aligned}$$

where $s_{it} = (y_t, y_{it})'$, $h = (1, 1)'$, $v_{it} = (\chi_t, \eta_{it})'$, and $\Sigma_v$ is diagonal with elements $\sigma_\chi^2$ and $\sigma_\eta^2$. The standard steady-state Kalman filter solution to the filtering problem implies that agent $i$'s time-$t$ forecast of $z_{t+j}$ follows the law of motion

$$\widetilde{E}_t^i [z_{t+j}] = \rho \widetilde{E}_{t-1}^i [z_{t-1+j}] + K \left(\rho^{j-1} s_{it} - h \widetilde{E}_{t-1}^i [z_{t-1+j}]\right),$$

where $K$ is the Kalman gain parameter, given by

$$\begin{aligned} K &= \rho \Sigma h' \left(h \Sigma h' + \Sigma_v\right)^{-1} \\ \Sigma &= \rho^2 \Sigma - \rho^2 \Sigma h' \left(h \Sigma h' + \Sigma_v\right)^{-1} h \Sigma + \Sigma_w. \end{aligned}$$

Denoting $\widetilde{E}_t [z_{t+j}]$ the cross-sectional average of the individual forecasts, we obtain

$$\widetilde{E}_t [z_{t+j}] = \rho \widetilde{E}_{t-1} [z_{t-1+j}] + K \left(\rho^{j-1} s_t - h \widetilde{E}_{t-1} [z_{t-1+j}]\right),$$

where $s_t = (z_t + \chi_t, z_t)'$. The law of motion for the average forecast can therefore be written as

$$\widetilde{E}_t [z_{t+j}] = \rho^{j-1} K h z_t + (\rho - Kh) \widetilde{E}_{t-1} [z_{t-1+j}] + \rho^{j-1} K_2 \chi_t,$$

where $K_2$ is the second element of $K$. Writing this forecast-updating equation in terms of forecast errors, we get

$$z_{t+j} - \widetilde{E}_t [z_{t+j}] = \left(\rho^j - \rho^{j-1} K h\right) z_t - (\rho - Kh) \widetilde{E}_{t-1} [z_{t-1+j}] - \rho^{j-1} K_2 \chi_t + \sum_{m=0}^{j-1} \rho^m w_{t+j-m}.$$

As in the sticky information model, this corresponds to regression (22) with $b_0 = 0$, $b_z = \left(\rho^j - \rho^{j-1} K h\right) \in [0, 1)$, and $b_f = -(\rho - Kh) \in (-1, 0]$. The regression coefficients reduce to $b_z = b_f = 0$ in the absence of signal noise ($\Sigma_v = 0$).

# I Equilibrium equations of the structural model

In this section, we summarize the full set of equilibrium conditions for the model described in Section 4.

## I.1 Representative household

Value function recursion:

$$V_t = (1-\beta)\log(C_t) - \beta\theta_t \log E_t\left[\exp\left(-\frac{1}{\theta_t}V_{t+1}\right)\right]$$

Budget constraint:

$$P_tC_t + B_{t+1} \leq (1-L_t)\,P_tD + L_tP_t\xi_t + R_{t-1}B_t - T_t$$

Stochastic discount factor:

$$s_{t+1} = \beta\frac{C_t}{C_{t+1}}$$

Euler equation for bond purchases:

$$1 = R_t\widetilde{E}_t\left[s_{t+1}\right]$$

## I.2 Labor market

Law of motion for employment:

$$L_t = (\rho + h_t)\,L_{t-1}$$

Hiring rate:

$$h_t = \frac{f_t\,(1-\rho L_{t-1})}{L_{t-1}}$$

Vacancy-filling rate:

$$q_t = \frac{h_t}{v_t}$$

Labor market tightness:

$$\zeta_t = \frac{v_tL_{t-1}}{1-\rho L_{t-1}}$$

Matching technology:

$$f_t = \mu\zeta_t^{\nu}$$

Present value of real wages (conditional on the job existing):

$$\xi_t^p = \xi_t + \rho\widetilde{E}_t\left[s_{t+1}\xi_{t+1}^p\right]$$

Present value of marginal revenue (conditional on the job existing):

$$\vartheta_t^p = \vartheta_t + \rho\widetilde{E}_t\left[s_{t+1}\vartheta_{t+1}^p\right]$$

The value of a job to the worker:

$$J_t^w = \xi_t^p + A_t$$

Outside benefits of being on a job:

$$A_t = (1-\rho)\,\widetilde{E}_t\left[s_{t+1}\left(f_{t+1}J_{t+1}^w + (1-f_{t+1})\,U_{t+1}\right)\right] + \rho\widetilde{E}_t\left[s_{t+1}A_{t+1}\right]$$

Present value of unemployment:

$$U_t = D + \widetilde{E}_t \left[ s_{t+1} \left( f_{t+1} J^w_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right]$$

Present value of the worker to the firm:

$$J_t = \vartheta^p_t - \xi^p_t$$

Free-entry condition:

$$J_t = \frac{\kappa_v}{q_t}$$

Nash bargaining surplus sharing rule for target wage:

$$\eta \left( J_t + \xi_t - \xi^*_t \right) = (1 - \eta) \left( J^w_t - U_t + \xi^*_t - \xi_t \right)$$

Actual wage:

$$\xi_t = \chi_w \xi_{t-1} + (1 - \chi_w) \xi^*_t$$

## I.3 Production

Optimal price setting:

$$\begin{aligned}
K_t &= \lambda \vartheta_t \frac{Y_t}{\exp(a_t)} + \chi_p \widetilde{E}_t \left[ s_{t+1} \pi^{\varepsilon}_{t+1} K_{t+1} \right] \\
F_t &= Y_t + \chi_p \widetilde{E}_t \left[ s_{t+1} \pi^{\varepsilon-1}_{t+1} F_{t+1} \right] \\
1 - \chi_p \pi^{\varepsilon-1}_t &= (1 - \chi_p) \left( \frac{K_t}{F_t} \right)^{1-\varepsilon}
\end{aligned}$$

## I.4 Shock processes and resource constraint

$\theta_t$ process:

$$\theta_t = (1 - \rho_\theta) \mu_\theta + \rho_\theta \theta_{t-1} + \sigma_\theta w^\theta_t$$

Technology process:

$$a_{t+1} = \rho_a a_t + \sigma_a w^a_{t+1}$$

Monetary policy rule:

$$\log \left( R_t / \overline{R} \right) = \rho_r \log \left( R_{t-1} / \overline{R} \right) + (1 - \rho_r) \left[ r_\pi \log \left( \pi_t / \overline{\pi} \right) + r_y \log \left( Y_t / Y^* \right) \right] + \sigma_r w^r_t$$

Aggregate resource constraint:

$$C_t + \frac{\kappa_v}{q_t} h_t L_{t-1} = Y_t$$

## References

Adam, Klaus and Sebastian Merkel. 2019. Stock Price Cycles and Business Cycles. CEPR Discussion Paper No. DP13866.

Adam, Klaus and Michael Woodford. 2012. Robustly Optimal Monetary Policy in a Microfounded New Keynesian Model. *Journal of Monetary Economics* 59 (5):468–487.

———. 2021. Robustly Optimal Monetary Policy in a New Keynesian Model with Housing. *Journal of Economic Theory* 198:105352.

Adam, Klaus, Albert Marcet, and Johannes Beutel. 2017. Stock Price Booms and Expected Capital Gains. *American Economic Review* 107 (8):2352–2408.

Adam, Klaus, Dmitry Matveev, and Stefan Nagel. 2021. Do Survey Expectations of Stock Returns Reflect Risk-Adjustments? *Journal of Monetary Economics* 117:723–740.

Alvarez, Fernando and Andy Atkeson. 2017. Random Risk Aversion and Liquidity: A Model of Asset Pricing and Trade Volumes. Unpublished manuscript.

Amromin, Gene and Steven A. Sharpe. 2014. From the Horse's Mouth: Economic Conditions and Investor Expectations of Risk and Return. *Management Science* 60 (4):845–866.

Angeletos, George-Marios, Fabrice Collard, and Harris Dellas. 2018. Quantifying Confidence. *Econometrica* 86 (5):1689–1726.

Angeletos, George-Marios, Zhen Huo, and Karthik A. Sastry. 2020. Imperfect Macroeconomic Expectations: Evidence and Theory. *NBER Macroeconomics Annual* 35 (1):1–86.

Armantier, Olivier, Wändi Bruine de Bruin, Simon Potter, Giorgio Topa, Wilbert van der Klaauw, and Basit Zafar. 2013. Measuring Inflation Expectations. *Annual Review of Economics* 5:273–301.

Armantier, Olivier, Giorgio Topa, Wilbert van der Klaauw, and Basit Zafar. 2017. An Overview of the Survey of Consumer Expectations. *Economic Policy Review* 23 (2):51–72.

Aruoba, S. Boragan, Luigi Bocola, and Frank Schorfheide. 2017. Assessing DSGE Model Nonlinearities. *Journal of Economic Dynamics and Control* 83:34–54.

Bacchetta, Philippe, Elmar Mertens, and Eric van Wincoop. 2009. Predictability in Financial Markets: What Do Survey Expectations Tell Us? *Journal of International Money and Finance* 28 (3):406–426.

Bachmann, Rüdiger, Steffen Elstner, and Eric R. Sims. 2013. Uncertainty and Economic Activity: Evidence from Business Survey Data. *American Economic Journal: Macroeconomics* 5 (2):217–249.

Baqaee, David Rezza. 2020. Asymmetric Inflation Expectations, Downward Rigidity of Wages, and Asymmetric Business Cycles. *Journal of Monetary Economics* 114:174–193.

Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer. 2015. X-CAPM: An Extrapolative Capital Asset Pricing Model. *Journal of Financial Economics* 115 (1):1–24.

Barsky, Robert B. and Eric R. Sims. 2011. News Shocks and Business Cycles. *Journal of Monetary Economics* 58 (3):273–289.

———. 2012. Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence. *American Economic Review* 102 (4):1343–1377.

Bassanin, Marzio, Ester Faia, and Valeria Patella. 2021. Ambiguity Attitudes, Leverage Cycle and Asset Prices. *Journal of International Economics* 129:103436.

Basu, Susanto and Brent Bundick. 2017. Uncertainty Shocks in a Model of Effective Demand. *Econometrica* 85 (3):937–958.

Basu, Susanto, Giacomo Candian, Ryan Chahrour, and Rosen Valchev. 2021. Risky Business Cycles. NBER Working Paper W28693.

Beaudry, Paul and Franck Portier. 2004. An Exploration into Pigou's Theory of Cycles. *Journal of Monetary Economics* 51 (6):1183–1216.

———. 2007. When Can Changes in Expectations Cause Business Cycle Fluctuations in Neoclassical Settings? *Journal of Economic Theory* 135 (1):458–477.

———. 2014. News-Driven Business Cycles: Insights and Challenges. *Journal of Economic Literature* 52 (4):993–1074.

Bhandari, Anmol, Jaroslav Borovička, and Paul Ho. 2016. Identifying Ambiguity Shocks in Business Cycle Models Using Survey Data. NBER Working Paper W22225.

Bianchi, Francesco, Cosmin L. Ilut, and Martin Schneider. 2018. Uncertainty Shocks, Asset Supply and Pricing over the Business Cycle. *Review of Economic Studies* 85 (2):810–854.

Bianchi, Francesco, Sydney Ludvigson, and Sai Ma. 2022. Belief Distortions and Macroeconomic Fluctuations. *American Economic Review* 112 (7):2269–2315.

Bidder, Rhys and Matthew E. Smith. 2012. Robust Animal Spirits. *Journal of Monetary Economics* 59 (8):738–750.

Blanchard, Olivier Jean and Charles M. Kahn. 1980. The Solution of Linear Difference Models under Rational Expectations. *Econometrica* 48 (5):1305–1312.

Blanchard, Olivier Jean, Jean-Paul L'Huillier, and Guido Lorenzoni. 2013. News, Noise, and Fluctuations: An Empirical Exploration. *American Economic Review* 103 (7):3045–3070.

Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer. 2020. Overreaction in Macroeconomic Expectations. *American Economic Review* 110 (9):2748–2782.

Borovička, Jaroslav and Katarína Borovičková. 2019. Risk Premia and Unemployment Fluctuations.

Borovička, Jaroslav and Lars Peter Hansen. 2014. Examining Macroeconomic Models through the Lens of Asset Pricing. *Journal of Econometrics* 183 (1):67–90.

Brunnermeier, Markus K. and Jonathan A. Parker. 2005. Optimal Expectations. *American Economic Review* 95 (4):1092–1118.

Calvo, Guillermo A. 1983. Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics* 12 (3):383–398.

Candia, Bernardo, Olivier Coibion, and Yuriy Gorodnichenko. 2020. Communication and the Beliefs of Economic Agents. In *Navigating the Decade Ahead: Implications for Monetary Policy.* Economic Policy Symposium (Jackson Hole, WY) Proceedings, Federal Reserve Bank of Kansas City.

Capistrán, Carlos and Allan Timmermann. 2009. Disagreement and Biases in Inflation Expectations. *Journal of Money, Credit and Banking* 41 (2–3):365–396.

Caplin, Andrew and John Leahy. 2019. Wishful Thinking. NBER Working Paper W25707.

Carlson, John A. and Michael Parkin. 1975. Inflation Expectations. *Economica* 42 (166):123–138.

Carroll, Christopher D. 2003. Macroeconomic Expectations of Households and Professional Forecasters. *Quarterly Journal of Economics* 118 (1):269–298.

Carroll, Christopher D., Edmund Crawley, Jiri Slacalek, Kiichi Tokuoka, and Matthew N. White. 2020. Sticky Expectations and Consumption Dynamics. *American Economic Journal: Macroeconomics* 12 (3):40–76.

Chahrour, Ryan and Kyle Jurado. 2018. News or Noise? The Missing Link. *American Economic Review* 108 (7):1702–1736.

Chahrour, Ryan and Robert Ulbricht. 2023. Robust Predictions for DSGE Models with Incomplete Information. *American Economic Journal: Macroeconomics* 15 (1):173–208.

Chib, Siddhartha and Edward Greenberg. 1996. Markov Chain Monte Carlo Simulation Methods in Econometrics. *Econometric Theory* 12 (3):409–431.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy* 113 (1):1–45.

Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin. 2011. DSGE Models for Monetary Policy Analysis. In *Handbook of Monetary Economics*, vol. 3A, edited by Benjamin M. Friedman and Michael Woodford, chap. 7, 285–367. Amsterdam, The Netherlands: North-Holland.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2016. Unemployment and Business Cycles. *Econometrica* 84 (4):1523–1569.

Coibion, Olivier and Yuriy Gorodnichenko. 2012. What Can Survey Forecasts Tell Us about Information Rigidities? *Journal of Political Economy* 120 (1):116–159.

———. 2015a. Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts. *American Economic Review* 105 (8):2644–2678.

———. 2015b. Is the Phillips Curve Alive and Well After All? Inflation Expectations and the Missing Disinflation. *American Economic Journal: Macroeconomics* 7 (1):197–232.

Coibion, Olivier, Yuriy Gorodnichenko, and Rupal Kamdar. 2018a. The Formation of Expectations, Inflation and the Phillips Curve. *Journal of Economic Literature* 56 (4):1447–1491.

Coibion, Olivier, Yuriy Gorodnichenko, and Saten Kumar. 2018b. How Do Firms Form Their Expectations? New Survey Evidence. *American Economic Review* 108 (9):2671–2713.

Constantinides, George M. and Darrell Duffie. 1996. Asset Pricing with Heterogeneous Consumers. *Journal of Political Economy* 104 (2):219–240.

Cox, John C., Jr. Ingersoll, Jonathan E., and Stephen A. Ross. 1985. A Theory of the Term Structure of Interest Rates. *Econometrica* 53 (2):385–407.

Crump, Richard K., Stefano Eusepi, Andrea Tambalotti, and Giorgio Topa. 2022. Subjective Intertemporal Substitution. *Journal of Monetary Economics* 126:118–133.

Das, Sreyoshi, Camelia M. Kuhnen, and Stefan Nagel. 2020. Socioeconomic Status and Macroeconomic Expectations. *Review of Financial Studies* 33 (1):395–432.

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Rafael Wouters. 2007. On the Fit of New Keynesian Models. *Journal of Business & Economic Statistics* 25 (2):123–143.

Dew-Becker, Ian. 2014. A Model of Time-Varying Risk Premia with Habits and Production. Unpublished manuscript.

Dominitz, Jeff and Charles F. Manski. 2004. How Should We Measure Consumer Confidence? *Journal of Economic Perspectives* 18 (2):51–66.

Dupor, Bill, Jing Han, and Yi-Chan Tsai. 2009. What Do Technology Shocks Tell Us About the New Keynesian Paradigm? *Journal of Monetary Economics* 56 (4):560–569.

Elliott, Graham, Ivana Komunjer, and Allan Timmermann. 2008. Biases in Macroeconomic Forecasts: Irrationality or Asymmetric Loss? *Journal of the European Economic Association* 6 (1):122–157.

Epstein, Larry G. and Martin Schneider. 2003. Recursive Multiple-Priors. *Journal of Economic Theory* 113 (1):1–31.

Epstein, Larry G. and Stanley E. Zin. 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57 (4):937–969.

Eusepi, Stefano and Bruce Preston. 2011. Expectations, Learning, and Business Cycle Fluctuations. *American Economic Review* 101 (6):2844–2872.

Fernald, John. 2014. A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. Federal Reserve Bank of San Francisco Working Paper No. 2012–19.

Fudenberg, Drew and David K. Levine. 1993. Self-Confirming Equilibrium. *Econometrica* 61 (3):523–545.

Gennaioli, Nicola, Yueran Ma, and Andrei Shleifer. 2015. Expectations and Investment. In *NBER Macroeconomics Annual 2015*, vol. 30, edited by Martin Eichenbaum and Jonathan A. Parker, 379–442. Chicago, IL: University of Chicago Press.

Gertler, Mark, Luca Sala, and Antonella Trigari. 2008. An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining. *Journal of Money, Credit and Banking* 40 (8):1713–1764.

Giglio, Stefano, Matteo Maggiori, Johannes Stroebel, and Stephen Utkus. 2021. Five Facts About Beliefs and Portfolios. *American Economic Review* 111 (5):1481–1522.

Gilboa, Itzhak and David Schmeidler. 1989. Maxmin Expected Utility with Non-Unique Prior. *Journal of Mathematical Economics* 18 (2):141–153.

Greenwood, Robin and Andrei Shleifer. 2014. Expectations of Returns and Expected Returns. *Review of Financial Studies* 27 (3):714–746.

Hall, Robert E. 2017. High Discounts and High Unemployment. *American Economic Review* 107 (2):305–330.

Hansen, Lars Peter and Thomas J. Sargent. 2001a. Acknowledging Misspecification in Macroeconomic Theory. *Review of Economic Dynamics* 4 (3):519–535.

———. 2001b. Robust Control and Model Uncertainty. *American Economic Review* 91 (2):60–66.

———. 2016. Sets of Models and Prices of Uncertainty. NBER Working Paper W22000.

Hansen, Lars Peter, Thomas J. Sargent, Gauhar Turmuhambetova, and Noah Williams. 2006. Robust Control and Model Misspecification. *Journal of Economic Theory* 128 (1):45–90.

Harrison, J. Michael and David M. Kreps. 1978. Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations. *Quarterly Journal of Economics* 92 (2):323–336.

Holmes, Mark H. 1995. *Introduction to Perturbation Methods*. New York: Springer.

Ichiue, Hibiki and Shusaku Nishiguchi. 2015. Inflation Expectations and Consumer Spending at the Zero Bound: Micro Evidence. *Economic Inquiry* 53 (2):1086–1107.

Ilut, Cosmin L. and Martin Schneider. 2014. Ambiguous Business Cycles. *American Economic Review* 104 (8):2368–2399.

Jordà, Òscar. 2005. Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review* 95 (1):161–182.

Jurado, Kyle. 2016. Advance Information and Distorted Beliefs in Macroeconomic and Financial Fluctuations. Unpublished manuscript.

Kamdar, Rupal. 2018. The Inattentative Consumer: Sentiment and Expectations. Unpublished manuscript.

Kehoe, Patrick, Pierlauro Lopez, Virgiliu Midrigan, and Elena Pastorino. 2023. Asset Prices and Unemployment Fluctuations: A Resolution of the Unemployment Volatility Puzzle. *Review of Economic Studies* 90 (3):1304–1357.

Kohlhas, Alexandre N. and Ansgar Walther. 2021. Asymmetric Attention. *American Economic Review* 111 (9):2879–2925.

Leduc, Sylvain and Zheng Liu. 2016. Uncertainty Shocks Are Aggregate Demand Shocks. *Journal of Monetary Economics* 82:20–35.

Li, Dake, Mikkel Plagborg-Møller, and Christian K. Wolf. 2021. Local Projections vs. VARs: Lessons From Thousands of DGPs. Working paper, Princeton University and University of Chicago.

Lombardo, Giovanni. 2010. On Approximating DSGE Models by Series Expansions. ECB Working Paper No. 1264.

Lucas, Robert E., Jr. 1972. Expectations and the Neutrality of Money. *Journal of Economic Theory* 4 (2):103–124.

Maćkowiak, Bartosz and Mirko Wiederholt. 2015. Business Cycle Dynamics under Rational Inattention. *Review of Economic Studies* 82 (4):1502–1532.

Maenhout, Pascal J., Andrea Vedolin, and Hao Xing. 2021. Robustness and Dynamic Sentiment.

Malmendier, Ulrike and Stefan Nagel. 2016. Learning from Inflation Experiences. *Quarterly Journal of Economics* 131 (1):53–87.

Mankiw, N. Gregory and Ricardo Reis. 2002. Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *Quarterly Journal of Economics* 117 (4):1295–1328.

———. 2007. Sticky Information in General Equilibrium. *Journal of the European Economic Association* 5 (2–3):603–613.

Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers. 2003. Disagreement about Inflation Expectations. In *NBER Macroeconomics Annual 2003*, vol. 18, edited by Mark Gertler and Kenneth Rogoff, 209–248. Cambridge, MA: MIT Press.

Manski, Charles F. 2017. Survey Measurement of Probabilistic Macroeconomic Expectations: Progress and Promise. In *NBER Macroeconomics Annual 2017*, vol. 32, edited by Martin Eichenbaum and Jonathan A. Parker, 411–471. Chicago, IL: University of Chicago Press.

Miranda-Agrippino, Silvia and Giovanni Ricco. 2021. The Transmission of Monetary Policy Shocks. *American Economic Journal: Macroeconomics* 13 (3):74–107.

Mitra, Indrajit and Yu Xu. 2023. Ambiguity and Unemployment Fluctuations. Working paper.

Molavi, Pooya. 2019. Macroeconomics with Learning and Misspecification: A General Theory and Applications. Unpublished manuscript.

Montiel Olea, José Luis and Mikkel Plagborg-Møller. 2021. Local Projection Inference is Simpler and More Robust Than You Think. *Econometrica* 89 (4):1789–1823.

Morris, Stephen. 1995. The Common Prior Assumption in Economic Theory. *Economics and Philosophy* 11 (2):227–253.

Nagel, Stefan and Zhengyang Xu. 2022. Asset Pricing with Fading Memory. *Review of Financial Studies* 35 (5):2190–2245.

Piazzesi, Monika, Juliana Salomao, and Martin Schneider. 2015. Trend and Cycle in Bond Premia. Unpublished manuscript.

Ravenna, Federico and Carl E. Walsh. 2008. Vacancies, Unemployment, and the Phillips Curve. *European Economic Review* 52 (8):1494–1521.

Rozsypal, Filip and Kathrin Schlafmann. 2019. Overpersistence Bias in Individual Income Expectations and its Aggregate Implications. Forthcoming in American Economic Journal: Macroeconomics.

Schmidt, Lawrence D.W. 2016. Climbing and Falling Off the Ladder: Asset Pricing Implications of Labor Market Event Risk. MIT Sloan Working Paper 5500-16.

Schmitt-Grohé, Stephanie and Martín Uribe. 2004. Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control* 28 (4):755–775.

Shiller, Robert J. 1997. Why do People Dislike Inflation? In *Reducing Inflation: Motivation and Strategy*, edited by Christina D. Romer and David H. Romer, chap. 1, 13–70. Chicago, IL: University of Chicago Press.

Shimer, Robert. 2005. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review* 95 (1):25–49.

———. 2010. *Labor Markets and Business Cycles*. Princeton, NJ: Princeton University Press.

Sims, Christopher A. 2002. Solving Rational Expectations Models. *Computational Economics* 20 (1–2):1–20.

———. 2003. Implications of Rational Inattention. *Journal of Monetary Economics* 50 (3):665–690.

Smets, Frank and Raf Wouters. 2007. Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach. *American Economic Review* 97 (3):586–606.

Strzalecki, Tomasz. 2011. Axiomatic Foundations of Multiplier Preferences. *Econometrica* 79 (1):47–73.

Szöke, Bálint. 2022. Estimating Robustness. *Journal of Economic Theory* 199:105225.

Tanaka, Mari, Nicholas Bloom, Joel M. David, and Maiko Koga. 2020. Firm Performance and Macro Forecast Accuracy. *Journal of Monetary Economics* 114:26–41.

Thomas, Lloyd B., Jr. 1999. Survey Measures of Expected U.S. Inflation. *Journal of Economic Perspectives* 13 (4):125–144.

Vellekoop, Nate and Mirko Wiederholt. 2019. Inflation Expectations and Choices of Households. Unpublished manuscript.

Woodford, Michael. 2003a. Imperfect Common Knowledge and the Effects of Monetary Policy. In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, edited by Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford, chap. 1, 25–63. Princeton, NJ: Princeton University Press.

———. 2003b. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

———. 2010. Robustly Optimal Monetary Policy with Near-Rational Expectations. *American Economic Review* 100 (1):274–303.

———. 2013. Macroeconomic Analysis Without the Rational Expectations Hypothesis. *Annual Review of Economics* 5 (1):303–346.

Zhao, Guihai. 2017. Confidence, Bond Risk, and Equity Returns. *Journal of Financial Economics* 126 (3):668–688.